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ANALYSIS AND IMPLICATIONS OF HIGH FREQUENCY RADIO DIRECTION FINDER BEARING STUDIES

N6-ori-71 Task XV
ONR Project No. 076 161
TECHNICAL REPORT NO. 20

RADIO DIRECTION FINDING SECTION
ELECTRICAL ENGINEERING RESEARCH LABORATORY
ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

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ANALYSIS AND IMPLICATIONS OF HIGH FREQUENCY RADIO DIRECTION FINDER BEARING STUDIES

N6-ori-7115 ONR Project No. 076-161

Technical Report No. 20

by

Albert D. Bailey

1 September 1954

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FOREWORD

This technical report is a partial reproduction of a Ph. D. thesis entitled, "An Investigation of the Direction of Arrival of Radio Waves". It goes beyond the scope of the thesis in that the results are compared with those of other investigators who have written classified reports. Because of this comparison, the report is classified.

ABSTRACT

A theoretical and experimental investigation was made of the direction of arrival of high frequency radio waves. Emphasis was placed upon the application of probability theory and statistics to known and proposed methods for improving the accuracy of the measurement of the direction of the wave source and decreasing the time required for such measurement. Means for reduction of the wave interference error was a specific goal.

Wave interference error due to multipath propagation is a significant contributor to the total standard deviation of the indicated bearing. It was shown from theoretical considerations that provided the probability distribution for the time phase of each of several interfering rays is uniform, the long-time mean error due to wave interference approaches zero. Also assuming that the wave polarization is random, the corresponding long-time mean error due to polarization approaches zero.

In order to investigate and experimentally verify the theoretical implications, an electronic bearing data computer and recorder was engineered. The computer-recorder samples the directional information data from a twin-channel cathode-ray type direction finder twenty-five times per second, computes the indicated bearing, and records the bearing and the signal amplitude on 35 millimeter film.

Experimental studies were made of the indicated bearings during those times of the day and year when wave interference effects would be most likely. Observations were made at Savoy, Illinois, on 6.42 mc/s propagations from Columbus, Ohio, and 5.0 mc/s propagation from WWV, Beltsville, Maryland.

The progressive or cumulative mean of the indicated bearings was considered the most significant of the several statistics that were calculated. The few isolated non-conforming cases are probably examples of lateral deviation effects. The improvement in accuracy due to time averaging over all data may be inferred from the following standard deviations of the indicated cumulative mean.

AVERAGING TIME	STANDARD DEVIATION OF THE CUMULATIVE MEAN
14 seconds	1.4°
1 minute	1.0°
2 minutes	0.7°
3 minutes	0.5°
4 minutes	0.4°

Censoring schemes that are premised upon using a strong signal or a strong and steady signal as a criterion for "good bearings" will generally yield an improvement in accuracy of the indicated bearing. However, the results of this investigation demonstrated that the cumulative mean of all bearings is the best estimate of the great circle bearing. Central-limit theorem behavior was exhibited in more than 95 percent of all cases studied.

A comparison of these results with those obtained on wide-base systems such as the KOMET, and the Naval Research Laboratory installation at Fox Ferry on the Potomac River indicates that the statistical accuracy figures stated for the several wide base systems may be approached as closely as one pleases with a small base system provided sufficient time is allowed to obtain a proper cumulative mean

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1. INTRODUCTION

1.1 Statement of the Problem

The avowed purpose of this effort was to investigate the direction of arrival of radio waves with primary emphasis upon the application of the theory of probability and statistics to known and proposed methods for increasing the accuracy of the measurement of the direction of the source of the wave and decreasing the time required for such a measurement. Practical means for the reduction of the wave interference error was a specific goal.

The motives underlying the investigation were prompted by certain observations which, when considered collectively, indicated a need for additional study of radio wave interference phenomena as it may effect the bearing error of small-base radio direction finders.

First, the radio direction finding research group at the University of Illinois had made some theoretical studies and simulated model investigations of the bearing errors that can arise in several radio direction finder systems under certain assumed conditions of radio wave interference. Thus it was possible to assign numbers to the magnitude of the radio wave interference error that would occur in a given radio direction finder under assumed and idealized conditions. However, it was not possible to state with certainty what was the best way of getting an accurate bearing in practice. One did not know how long a period was needed for sampling an arriving set of radio waves and how many samples were needed to determine a wave-interference error free bearing.

Second, it had been observed in the results of certain investigations performed by others within the last decade that the probable bearing deviation can be made considerably smaller than the two to five degree figure of the conventional Adcock system. The improvement is primarily due to a considerable increase in the wave collector base dimensions and number of active elements in the collector. Costwise this improvement is expensive: compared to an Adcock system, a tenfold increase in the cost of such an installation may easily result. Because of the extensive cost increase, one is led to inquire into the possibility of controlling the magnitude of the Adcock bearing error by

proper treatment of the data. Hence the wave interference error and the polarization error may possibly be reduced without entailing too great an increased cost.

Third, the radio direction finding research group at the University of Illinois has completed the installation of a high quality dual-channel type radio direction finder on a very good site near Savoy, Illinois. The equipment is capable of giving continuous and instantaneous bearing indications. Thus there is available a practical means for verifying improvement schemes that seem to have potential merit.

Fourth, in order to best use the direction finder there was needed an unbiased analog data computer and recorder that would be free of observational error, and which would record data fast enough to preserve its continuity, i. e., all fluctuations of the data should be recorded. A speed of data recording of at least one-hundred times faster than that of the human observer is needed. The modern electronic digital computer permits extremely rapid functional analyses of data provided the data are in a convenient form for instructing the computer. This convenience of proper analog-to-digital data conversion should be a prime consideration in the development of any modern bearing data computer and recorder and was indeed an added incentive in this investigation.

1.2 Method of Attack

The method of attack was two-fold; it consisted of a theoretical study followed by an experimental investigation. The theoretical study consisted of acquiring a background in the theory of probability and statistics, reviewing the pertinent works of other investigators, and formulating some positive approaches toward the solution of the problem. The experimental investigation began with the conception, design, construction, and application of measuring facilities and techniques that provided data for the statistical frequency distributions, averages, variances, and correlations with other variables under the several known and-or controlled causal conditions of bearing error occurrence.

By appropriate analyses of the statistics of the reduced data it should be possible to infer what limits in the improvement of bearing accuracy and precision together with decrease in measuring time are feasible in the light of present engineering practice. A comparison

of the conditional accuracy figures so obtained with those of existing and proposed wide-base radio direction finders should provide an additional quantitative basis for making logical decisions in the choice between wide- or narrow-base direction finding systems for specified applications.

2. PRELIMINARY CONSIDERATIONS

2.1 Definitions

It seems logical and convenient to consider at the outset certain definitions and principles that are necessary to the understanding of the main body of the investigation. Accordingly, several definitions are given for pertinent aspects of the radio direction finding problem.

Direction Finder.--"A direction finder is a radio receiving device which permits determination of the line of travel of radio waves as received."[1]

Sense Finder.--"A sense finder is that portion of a direction finder which permits determination of direction without one-hundred-eighty degree ambiguity."[2]

Radio direction finder.--"A radio direction finder may be defined as a device for determining the direction of arrival of radio-frequency energy. It is a receiving system and operates on the energy that it extracts from the passing radio waves."[3]

Direction finder network.--A direction finder network consists of two or more direction finders suitably oriented in geometrical relationship with one another to permit determination of the most probable location of the source of a radio frequency signal.

Radio direction finding problem.--The radio direction finding problem may be posed as follows: given the indicated direction of arrival of radio frequency energy as received at two or more geographically known locations, determine the geographical location of the source of energy.

Radio direction finding research.--"The object of all research in radio direction finding is, on the one hand, to try to find the causes of errors and eliminate them and, on the other, to lay down conditions of use of the instrument which will minimize any residual errors."[4]

1. American Institute of Electrical Engineers, *American Standard Definition of Electrical Terms*, (New York: 33 West Thirty-Ninth Street, 1942).
2. *Ibid.*
3. Radio Research Laboratory Staff of Harvard University, *Very High Frequency Techniques*, (New York: McGrew-Hill Co. Ind., 1947) Vol. I, p. 199.
4. A.S.D. 031 Direction Finding - No S1/D.F.I., "*Limits of Use of the Cross Buried-U Adcock High-Frequency Cathode-Ray Direction Finder*," Air Force Head-Quarters, Melbourne, Australia, S.C.I October, 1942. (CONFIDENTIAL)

Thus, research aims first to eliminate and/or reduce the *causes* of error and second, to minimize, in one way or another, the *effects* of residual causes. It is the latter aim that is of principal concern in this investigation.

By way of discussion it is seen that the implication of the last definition goes beyond the scope of the earlier definitions and it suggests that a good direction finder in the sense of the earlier definitions may be a very deficient instrument for the solution of the radio direction finding problem. As an example, if the polarization of the received wave is different from that on which the design of the instrument was premised, a bearing error may occur. As will be seen later, there are several sources of error that may intervene along the line of travel between the signal source and the direction finder.

2.2 Components of Error in Direction of Arrival Measurements

All physical measurements are subject to error, and in particular the radio direction finder instrument is subject to several error causing agents. If one asks for the ultimate answer in direction finding, then several additional error causing mechanisms must be considered. It has been found convenient to divide the basic causes of error into the several categories of (a) instrumental error, (b) site error, (c) propagation error, (d) source error, and (e) operator error.

2.3 Independence of Errors

Each of the several causes of errors stated above is assumed to be an independent cause. The fact that there is no logical reason for thinking otherwise is not a justification for such an assumption. However, the experimental work that has been done to date lends support to this assumption as a tenable one. It is a common assumption in the present day treatment of direction finder errors.[5]

2.4 Means for Error Reduction

The previous discussions have pointed up the fact that a radio direction finder bearing observation is subject to several basic error producing causes. In fact one may safely say that any single bearing observation is not to be trusted. Obviously one needs to know more about the character of the source, the transmission paths, the direction finder site and the instrument itself at the time that the bearing is

5. W. Ross, "The Estimation of the Probable Accuracy of High-Frequency Radio Direction-Finding Bearing" *Journal of the Institution of Electrical Engineers* (London) Part III A (Radio Communication) 1947, *Vol. 94, No. 15*, p. 723.

taken. However, if all this were known, there would be little need for the direction finder. In lieu of the above, one presently utilizes the methods of statistics. The procedure is simply to observe the bearing for a sufficient period of time to get a "representative random sample." The mean of this sample -- after removal of the systematic errors -- is the best estimate of the true bearing. Further, if normal law behavior is assumed, the standard deviation of the sample can be calculated, and confidence limits can be established as to the likelihood of the source being in any designated sector. An objection to the statistical method is that it takes time, and time is often a precious commodity in direction finding.

A third possible way of reducing the error and one which is a part of the subject of this dissertation is to attempt to recognize the times at which the bearing is subject to appreciable error and to prevent the taking of bearings at such times. The bearings that are then obtained are supposedly free of serious error and it is hoped that one single bearing so obtained may be as good as a long-time average. The worth of the method depends upon the fact that the indicated bearing fluctuates that in the course of the fluctuation the indicated bearing coincides with the great circle line of bearing, that one has a means for identifying the time of coincidence, and that this case occurs frequently enough so that in any observational period one gets at least one cut -- the last two conditional requirements need verification.

A fourth possible way of reducing the error is in the use of either large aperture systems or ensembles of small aperture systems. Each of the above permits averaging of the wave fronts over a wide base. The former are less susceptible to polarization error and large wave interference errors. The success of the latter depends upon the assumption that the time series of the indicated bearings is stationary in the probability sense. Either system should mitigate the site error.

2.5 The Ultimate Limit of Accuracy in the Determination of the Great Circle Bearing of High Frequency Radio Transmitters

Lateral deviation of sky-reflected waves is probably the principal factor precluding any further improvement in short-time bearing accuracy after the elimination of the instrumental, site and wave interference errors.

A quantitative estimate of the lateral deviation of a sky-reflected wave due to the tilt or slope of an ionospheric layer is afforded by Fig. 1.

In the course of the last twenty-five years many papers have appeared that discuss the limiting accuracy in the determination of the great circle bearing of a high-frequency signal source. The work of Friis, Feldman and others in America; the work of Eckersley, Barfield, Ross and others in England; the work of Crone, Weber and others in Germany have verified and established the limitations in the quantitative manner that is summarized in Fig. 1.[6-15]

The measuring equipments that were used in establishing the limitations were specialized research tools incorporating large-base collector systems and were greatly restricted in azimuth -- generally the systems were pre-oriented for a particular great-circle bearing path. Consequently the systems were not attractive to radio direction finding since one usually requires unrestricted azimuthal performance.

6. H. T. Friis, "Oscillographic Observations on the Direction of Propagation and Fading of Short Waves," *Proc. IRE*, 1928, *Vol. 16*, pp. 658-685.

7. H. T. Friis, C. B. Feldman and W. N. Sharpless, "The Determination of the Direction of Arrival of Short Radio Waves," *Proc. IRE*, 1934, *Vol. 22*, pp. 47-78.

8. C. B. Feldman, "Deviations of Short Radio Waves from the London-New York Great-Circle Path," *Proc. IRE*, 1939, *Vol. 27*, pp. 635-645.

9. T. L. Eckersley, "An Investigation of Short Waves," *Marconi Review*, 1929, *Vol. 1, No. 8* pp. 13-23.

10. T. L. Eckersley, "Scattering, Polarization Errors, and the Accuracy of Short Wave Direction Finding," *Marconi Review*, 1935. *Vol. 5, No. 53*, pp. 1-8.

11. R. H. Barfield and W. Ross, "The Measurement of the Lateral Deviation of Radio Waves by Means of a Spaced-Loop Direction Finder," *Journal Inst. Elect. Engrs.*, 1938, *Vol. 83*, pp. 98-110.

12. W. Ross, *Lateral Deviation of Radio Waves Reflected at the Ionosphere*, Special Report no 19 of the Department of Scientific and Industrial Research, Radio Research (London: HMSO 1949).

13. W. Ross, and E. N. Bramley, "Measurements of the Direction of Arrival of Short Waves Reflected at the Ionosphere," *Proceedings of the Royal Society* (London: 1951) *Vol. 207, No. 1089*, pp. 251-267.

14. W. Crone, "Possibilities and Limitations of Direction Finding with Sky Waves," Paper no. 4 of *Radio Direction Finding and Navigational Aids*. Special Report No. 21 of the Department of Scientific and Industrial Research, Radio Research, (London: HMSO, 1951) pp. 34-58.

15. R. Weber, "The Results of Measurements Made on the 2-Base 'Komet' Installation at Ismaning," paper No. 5, *Ibid*.

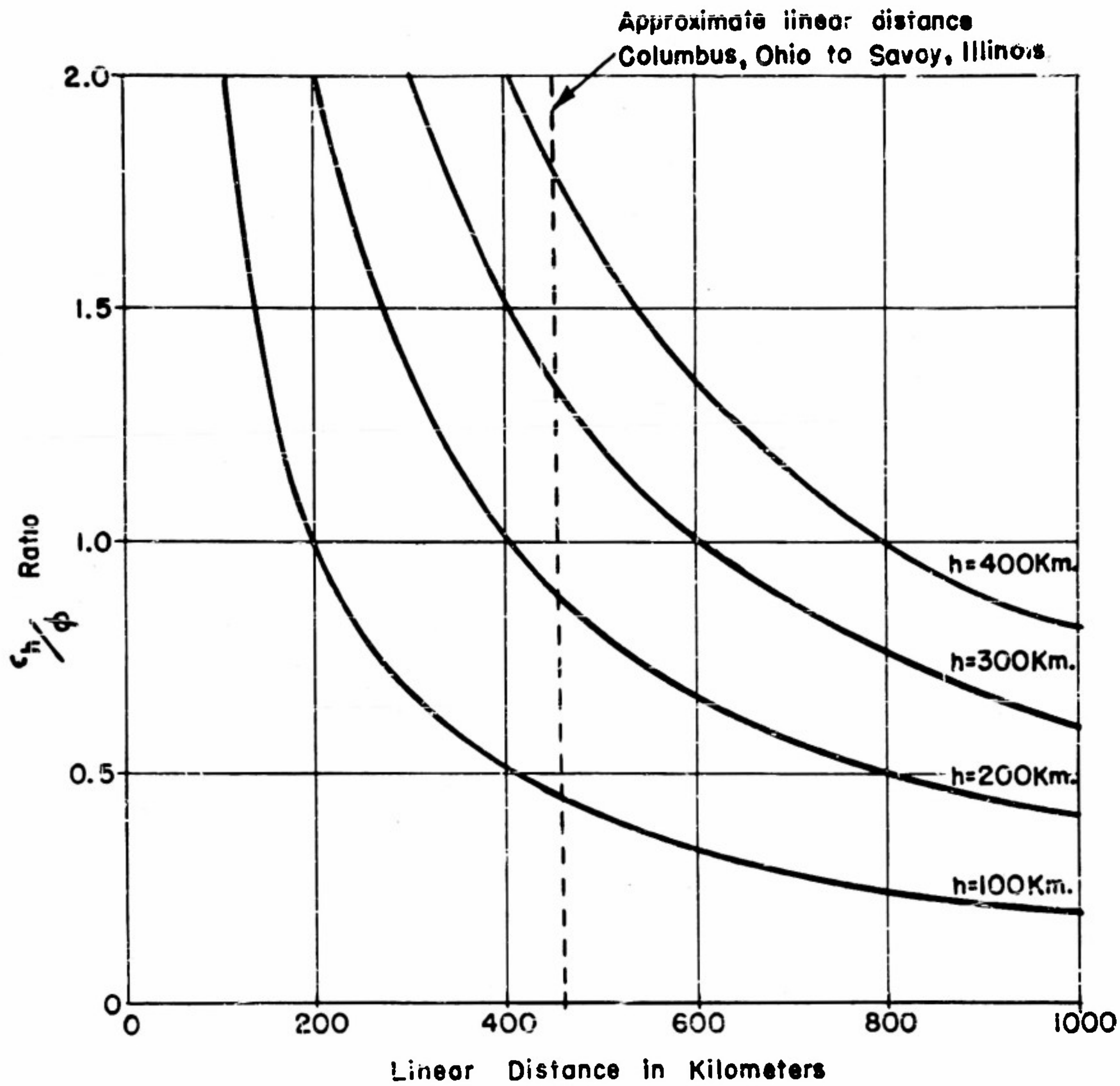


FIGURE 1 RATIO OF BEARING DEVIATION TO THE SLOPE OF THE IONOSPHERE AS A FUNCTION OF EFFECTIVE LAYER HEIGHT AND LINEAR DISTANCE -- A FLAT EARTH AND MAXIMUM TILT EFFECT ARE ASSUMED.

The practical limitation in accuracy of small-base radio direction finders has been summarized from time to time. A rather recent paper by J. A. Pierce gives an average angular error for an Adcock system of two degrees for high frequency working and one degree for low frequency and ground wave working.[16] The curves of Fig. 2 were also given to compare the accuracy versus range performance of navigation systems using the Loran principle and conventional radio direction finders. It is of interest to state that the case for the accuracy of the radio direction finder can be made considerably stronger by proper statistical treatment of the bearing data. This will be made evident by the experimental evidence that follows.

By way of comparison with Pierce's curves, a report prepared by the Australian Air Force in 1942 gives experimental performance figures on a good high frequency, twin-channel cathode-ray direction finder.[17] The results are summarized in Fig. 3 and are considered typical of short-time bearing accuracy that may be achieved by such systems.

Various proposals for the improvement in radio direction finder performance have been made from time to time. Pulse transmissions were first used in direction-of-arrival measurements in this country by Friis, Feldman, and Sharpless prior to 1934.[17] Pulse techniques in direction-of-arrival measurements were used by Eckersley and Smith prior to 1932.[18] This work was followed by papers and patents suggesting the use of pulse transmissions to reduce bearing error -- the general idea being that the first pulse to arrive would have come by the most direct path and hence would be least susceptible to error.[19-22] Runge patented the use of an auxiliary antenna that was susceptible to polarization and wave-interference fading effects to block the bearing

16. J. A. Pierce. "Electronic Aids to Navigation," *Advances in Electronics* (New York: Academic Press Inc., 1948), *Vol. I*, pp. 425-451.

17. See reference no. 4.

18. T. L. Eckersley and S. B. Smith, *British Patent No. 397524*, Marconi Wireless Telegraph Co., Ltd., 1932.

19. T. L. Eckersley, "Elimination of Night Effect with a Pulse Transmitter," *Marconi Review* (1934) *Vol. 46*, pp. 12-16.

20. Chireix, "Radio Direction-Finding Arrangement Free from Night Error," *Rev. Gen. d'El.* (1935), *Vol. 38*, P. 208D.

21. H. Plendl, "Direction Finding by Pulses." *Hochfrequenztechnik und Electroakustik*, (1937) *Vol. 50*, pp. 37-41.

22. F. Johnske, "Equipment for Direction-Finding Free from Night Effect", *Hockfrequenztechnik und Electroakustik*, (1941) *Vol. 58*, p. 46.

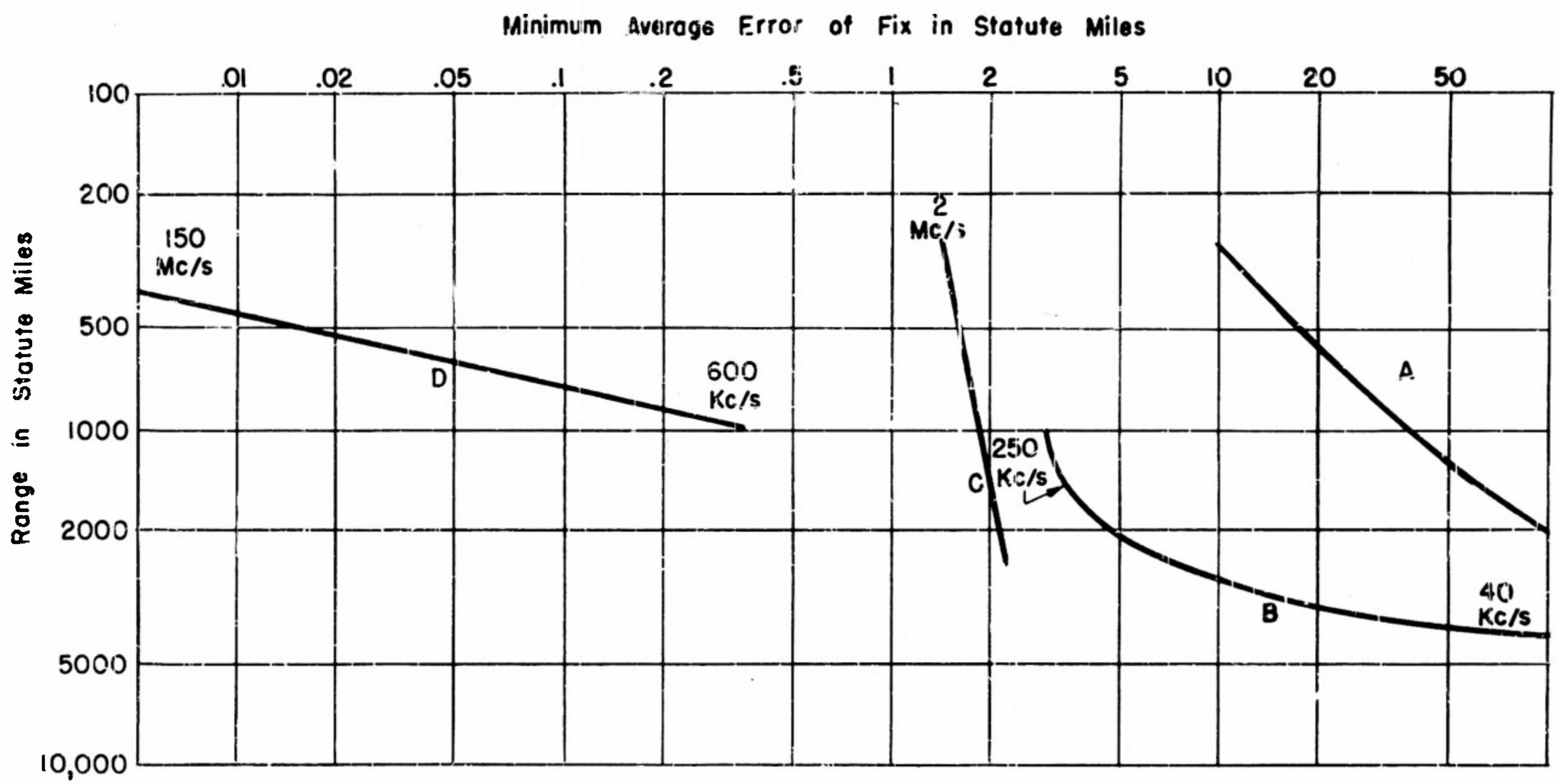


FIGURE 2 COMPARATIVE PERFORMANCE OF RADIO DIRECTION FINDING AND LORAN TYPE SYSTEMS (AFTER J. A. PIERCE).

A -- radio direction finding
B -- low frequency loran
C -- standard loran
D -- sky-wave loran

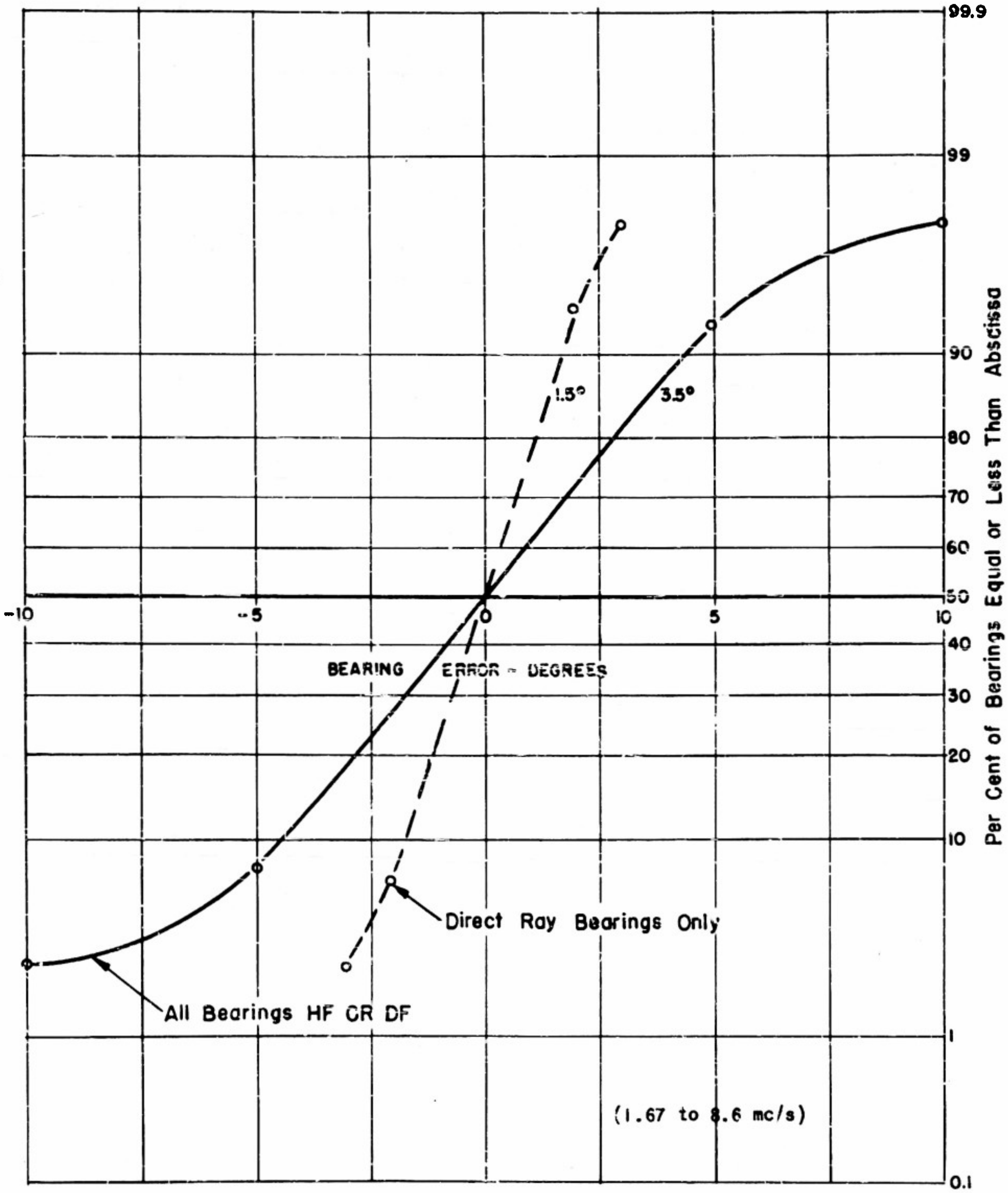


FIGURE 3 SUMMARY OF THE EXPERIMENTAL PERFORMANCE OF THE SHORT-TIME BEARING ACCURACY OF A HIGH-FREQUENCY TWIN-CHANNEL CATHODE-RAY DIRECTION FINDER (FROM REFERENCE 4)

indication when the error was supposedly pronounced.[23] Terman and Pettit described a compensated-loop direction finder that could be made free of polarization under very restricted conditions by proper adjustment.[24] These are only a few of the many schemes that have been proposed. Each has its potential merit but is only a partial solution at best.

23. W. Runge, "Direction Finding Free from Night Error," *Hochfrequenztechnik und Electroakustik*, (1945) *Vol. 60*, p. 177.

24. F. E. Terman and J. M. Pettit, "The Compensated Loop Direction Finder," *Proc. IRE*, (1945) *Vol. 33*, p. 12.

3. APPLICATION OF PROBABILITY THEORY TO THE WAVE INTERFERENCE PROBLEM

3.1 Wave Interference Bearing Error Formula

It is possible to derive a wave interference bearing error formula for a given radio direction finder provided sufficient assumptions are made. As an example of this, consider an idealized twin-channel, cathode-ray direction finder incorporating a narrow base crossed-Adcock collector, where by idealized is meant freedom from all error other than that under discussion. Let it be required to find the bearing error under the condition that N signals of the same frequency, and each linearly polarized in its own plane of incidence, but of arbitrary amplitude, arbitrary time phase, arbitrary azimuth, and arbitrary elevation are incident simultaneously upon the collector. The expression for the bearing error of the major-axis of the ellipse seen on the indicator screen can be shown to be:

$$
\frac{\dfrac{\sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Sin}(\phi_n + \gamma_n)}{1 + \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Cos}(\phi_n + \gamma_n)} + \dfrac{\sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Sin}(\phi_n - \gamma_n)}{1 + \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Cos}(\phi_n - \gamma_n)}}{1 - \dfrac{\sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Sin}(\phi_n + \gamma_n) \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Sin}(\phi_n - \gamma_n)}{\begin{gathered}1 + \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Cos}(\phi_n + \gamma_n) + \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Cos}(\phi_n - \gamma_n) \\ + \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Cos}(\phi_n + \gamma_n) \sum_{2}^{N} h_n \left(\dfrac{\mathrm{Sin}\,\theta_n}{\mathrm{Sin}\,\theta_1}\right)^2 \mathrm{Cos}(\phi_n - \gamma_n)\end{gathered}}}
$$

where h_n, ϕ_n, θ_n, and γ_n are the amplitude, azimuth, incidence, and time phase respectively of the nth signal relative to the first signal.

In functional notation the wave interference error is

$$\varepsilon = f(h_n, \phi_n, \theta_n, \gamma_n)$$

where the symbols correspond in the order given to the variables discussed in order above. Now this is a multivalued function in that ε is not unique for a given set of values even though it is assumed that the variables are completely independent. Hence as it stands one has an interesting but rather useless formula since generally not enough is known about propagation conditions at any one time and place to calculate the corresponding error. Particularly is this true in the radio direction finding problem since the aim is to locate the origin of a transmission, but in order to completely evaluate the wave interference error one needs to know among other things the location in advance.

If one were in possession of sufficient statistical data, say the most probable values of h_n, ϕ_n, θ_n, and γ_n, the most probable error could be estimated. In fact if one were in possession of an error function of γ alone, then assuming that all time phases are equally likely (the uniform probability distribution) a root-mean - square error may be calculated. Thus given some $\varepsilon(\gamma)$ then one can calculate an expectation of the error and the square error as follows:

$$E[\varepsilon(\gamma)] = \int_0^{2\pi} \frac{\varepsilon(\gamma)d\gamma}{2\pi}$$

$$E[\varepsilon^2(\gamma)] = \int_0^{2\pi} \frac{\varepsilon^2(\gamma)d\gamma}{2\pi}$$

Error curves that are functions of γ alone have been calculated and will be considered below.

3.2 Expectation of Wave Interference Error

The theory of probability provides a theorem for the expectation of the mth moment of a function of several independent random variables

when the separate probability densities are known, thus:

$$E(\varepsilon^m) = \iiint \cdots \int [\varepsilon^m(r,s,t\cdots z)]\ (r)(s)(t)\cdots(z)\cdot drdsdt\cdots dz$$

By virtue of this theorem one may calculate a first moment of error or mean error and also the second moment of error about the mean or variance. The mathematical evaluation of this integral is not an attractive task since the wave interference error function is quite involved. However, a quantitative argument can be made for asserting that the theoretical expectation of the bearing error due to wave interference is zero. Consider again the wave interference bearing error formula given earlier. One notes the occurrence of terms of the form $\sum^N K_n \text{ Cos } x_n$ and/or $\sum^N K_n \text{ Sin } x_n$. If x is uniformly distributed, then for large N all sums are small and in the limit as $N \rightarrow \infty$ all sums approach zero. It then follows that the bearing error formula yields errors approaching zero for large N. That the mean error should be zero is borne out in practical direction finders, in fact radio direction finding practice is tacitly premised upon the statistical proposition that the mean value of the deviation of a large number of bearings is zero. On the other hand the variance is not zero and an estimate of this value is of paramount importance in evaluating the statistical performance of a radio direction finder under wave interference conditions. Hence the problem becomes one of somehow getting an estimate of the second moment of error.

3.3 Limiting Root-Mean-Square Deviation

Inspection of the wave interference error function given earlier reveals that elevation angle enters in such a way as to reduce the amplitude of an interfering ray. If one assumes that the signal having the lowest angle of elevation is the "desired" signal then the amplitude of each interfering signal is reduced by the factor $\left(\frac{\text{Sin } \theta_i}{\text{Sin } \theta_1}\right)^2$ where θ_i is the angle of incidence of the ith interfering signal. If it is further assumed for purposes of analysis that all signals arrive from

the same angle of elevation, the error function is simplified to some extent and the resulting error function is more pessimistic.

For the case of two signals the wave interference error function can then be shown to be

$$\varepsilon = 1/2\left\{\tan^{-1}\frac{h\ \mathrm{Sin}[(\phi_2-\phi_1)+(\gamma_2-\gamma_1)]}{1+h\ \mathrm{Cos}[(\phi_2-\phi_1)+(\gamma_2-\gamma_1)]}+\tan^{-1}\frac{h\ \mathrm{Sin}[(\phi_2-\phi_1)-(\gamma_2-\gamma_1)]}{1+h\ \mathrm{Cos}[(\phi_2-\phi_1)-(\gamma_2-\gamma_1)]}\right\}$$

Error curves as functions of time phase difference for one selected azimuthal separation and several amplitude ratios are given in Fig. 4. It will be observed that the mean error relative to the stronger signal is zero when averaged over the complete interval for any given ϕ and h. The rectangular (dashed) curves that correspond to h ratios of zero, one, and infinity form an enclosure that contains the majority of the bearing error cases. In fact, if one uses the rectangular outline to estimate the variance of the bearing error he will determine a number which is greater than the actual variance in any case. The estimate of an upper bound on the variance is a very simple calculation indeed when based upon the rectangular outline.

Figure 5 is an example of the limiting bearing error envelope geometry for a two signal interference case having an azimuthal separation of ϕ degrees. It can be shown that for $h \leq 1$ the variance is always less than 45ϕ square degrees. This implies that the indicated bearings are within the shaded portion. Further, if one can censor the data so as to avoid the times of greatest bearing swing, the variance can be made less than $\phi/2$ square degrees. This censoring can be done with ease in the case of two signals at least because there is a direct correlation between amplitude and time phase difference.

Figure 6 shows the square-root of the limiting variance as a function of angular separation between two interfering signals; the great reduction in limiting variance due to censoring is immediately evident. The possibility of doing this practically as a means of reducing the wave interference error effect is very attractive. It is of course made use of to some extent in present day direction finding. The censoring aspect will be considered later in more detail.

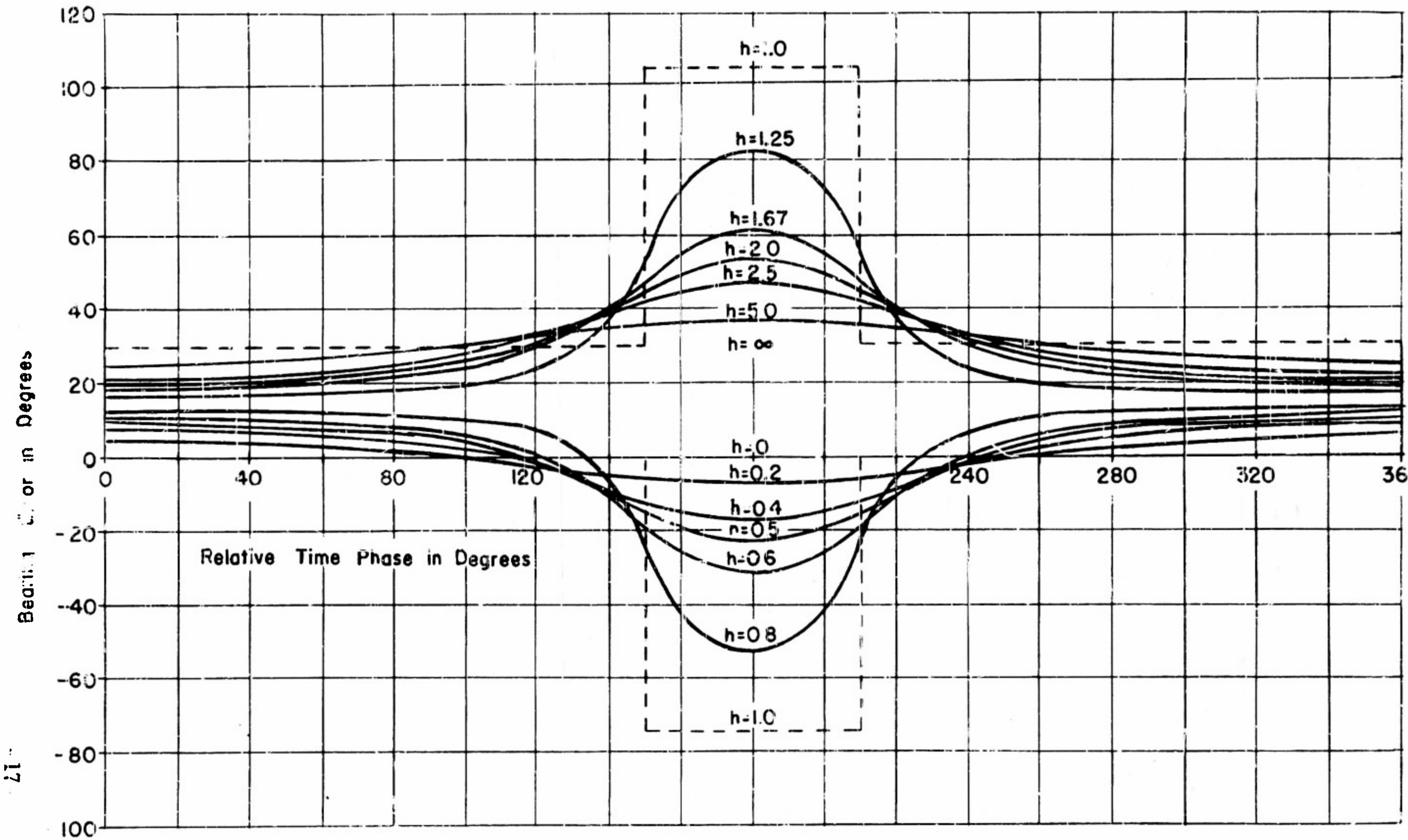


FIGURE 4 WAVE INTERFERENCE BEARING ERROR AS A FUNCTION OF TIME PHASE DIFFERENCE AND RELATIVE AMPLITUDE FOR TWO SIGNALS OF THE SAME FREQUENCY ARRIVING FROM DIRECTIONS HAVING A 30 DEGREE AZIMUTHAL SEPARATION.

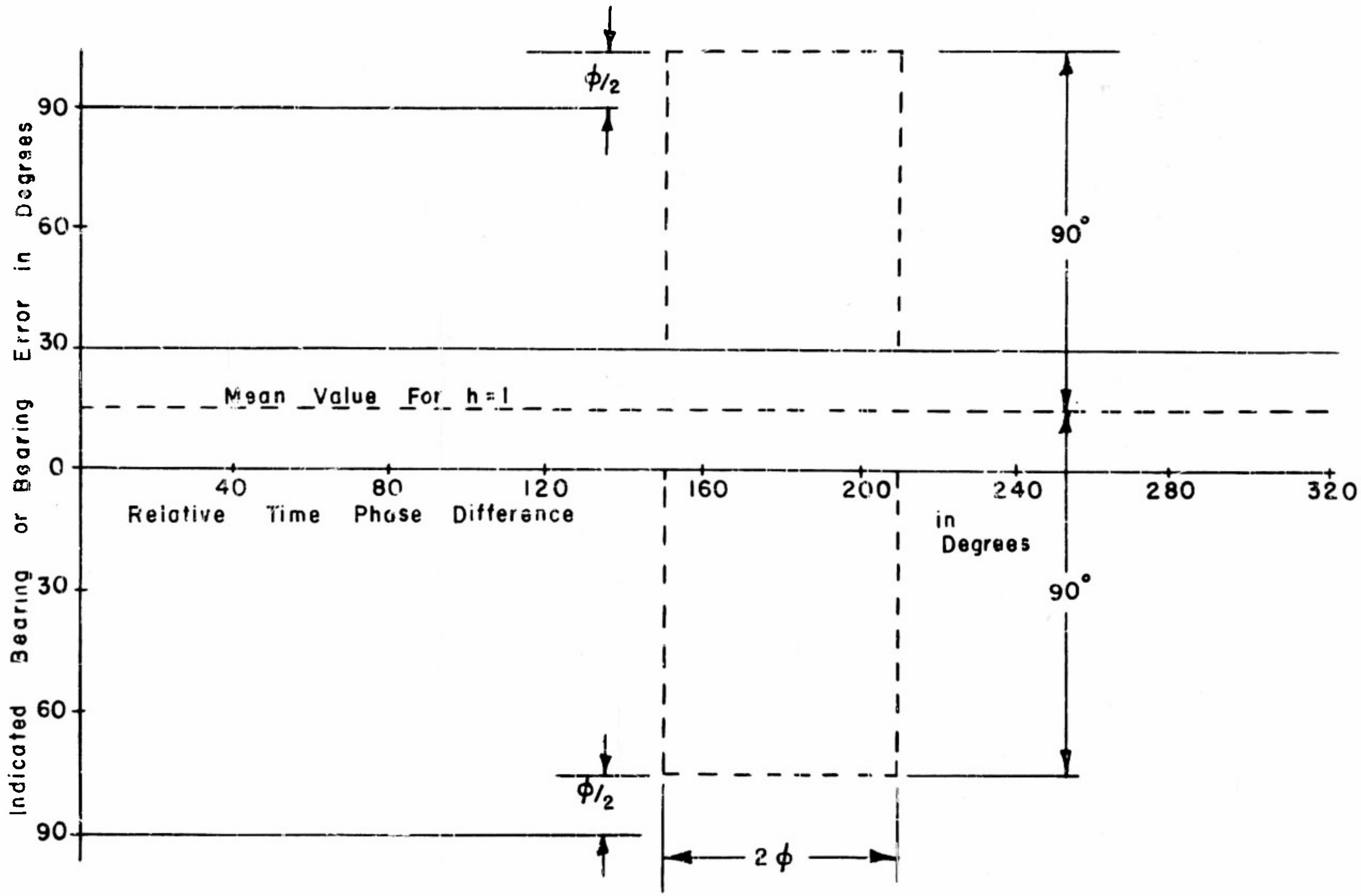


FIGURE 5 LIMITING BEARING ENVELOPE GEOMETRY.

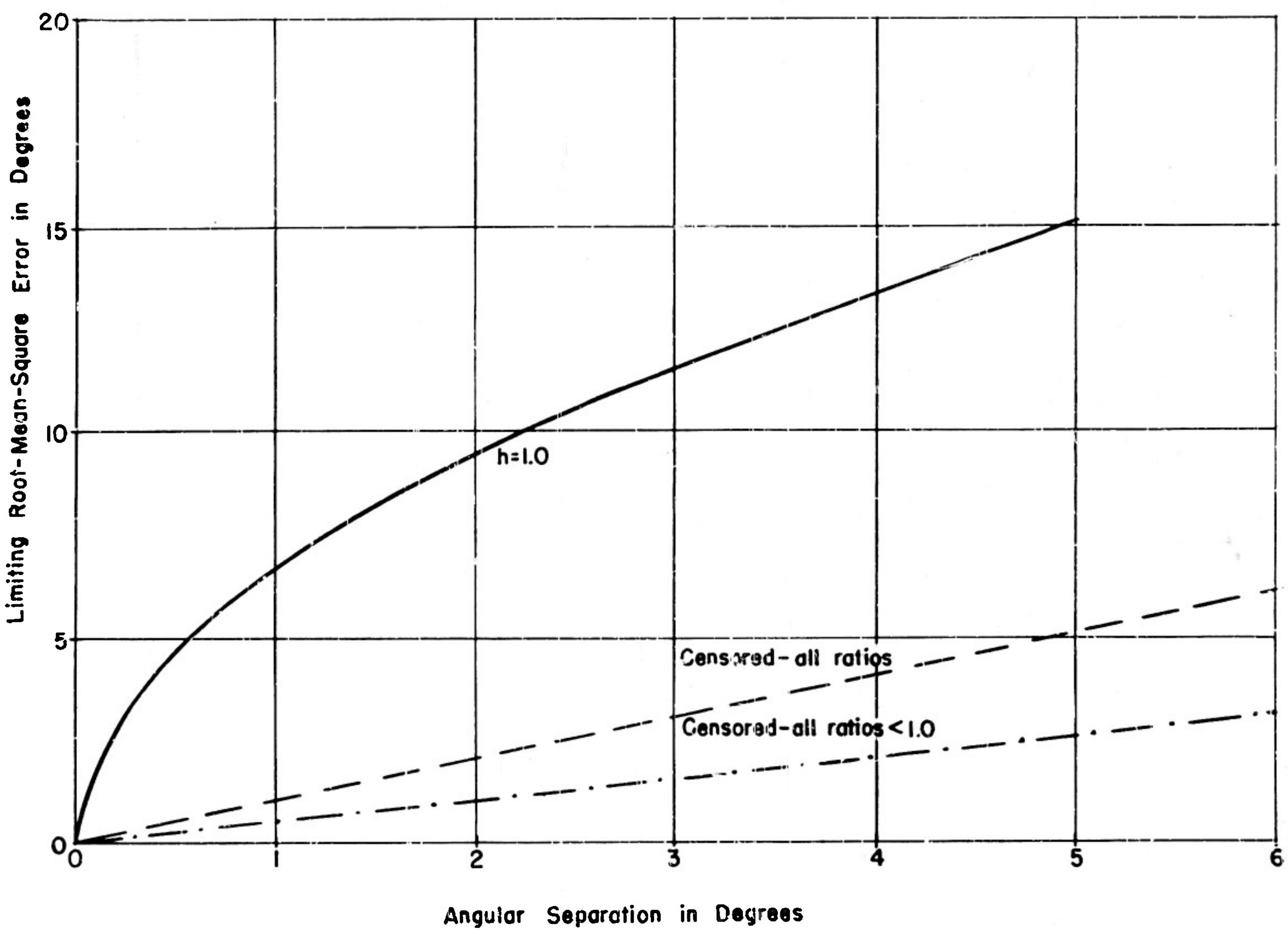


FIGURE 6 SOME UPPER BOUNDS ON ROOT-MEAN-SQUARE ERROR.

The next logical case that arises is that of three or more interfering signals. Horner has investigated the probable error under the set of conditions where a random signal composed of a large number n of small amplitude signals arriving from a narrow range of azimuth interferes with a desired signal.[25] It is shown that the probable error, i.e., the error for which the odds of it being exceeded are one-to-one, is $0.477\sqrt{n}$ times the maximum error caused by a single interfering ray. Now the maximum error caused by a single interfering ray is a simple matter to compute. Hence one has here a first step in the solution of the "direct" bearing error problem under conditions of multipath arrival. Because the stated condition does occur quite frequently in direction finding, one could use the $\sqrt{n}$ criterion as an estimator of the probable root-mean-square error for any specified number of interfering signals. However, in general n is not precisely known in advance, hence one cannot justify any practical necessity for greater precision in the estimator.

Statistics imply the operations of sampling and reduction of data to find characteristic numbers that will compactly describe the underlying population. It is now appropriate to consider the experimental phase of the problem in which certain questions are put to nature and answers are sought in the measureable behavior of natural phenomena. Out of these answers one can decide whether or not his theory is applicable and adequate, and if not, then find what further extensions in theory are necessary to reconcile his experimental results.

25. F. Horner, "A Problem in the Summation of Simple Harmonic Functions of the Same Amplitude and Frequency but of Random Phase," *Phil. Mag., Ser. 7, Vol. XXXVII*, March (1946), 145-162.

4. THE BEARING DATA RECORDER

4.1 The Sources of Available Data

The radio direction finder installation at the University of Illinois Airport near Savoy, Illinois is equipped to provide the following:

1. Directional pair voltages at 455 kcs from the N-S and E-W intermediate frequency amplifiers of the high frequency radio direction finder.
2. Bearing ellipse data presentation on a 5-inch cathode-ray tube screen.
3. A-scope presentation of the envelope of the received signal on one trace of a 5-inch dual-beam cathode-ray tube screen. The second trace of the cathode-ray screen is used to display the position of a variable length gating pedestal that permits selection of any portion of the A-scope presentation for independent study.
4. Wave polarization ellipse data presentation on the screen of a third 5-inch cathode-ray tube.
5. Photographic recording of the four displays on 16mm film at the rates of 1 and 5 frames per second.
6. Synchronization, variable width-and-delay rectangular gating and/or pedestal pulses from the electrical and photographic recording equipment.

In addition to the above there was needed an equipment that would efficiently sample the bearing and signal amplitude data at a more rapid rate and present the data in a manner that is readily amenable to statistical analysis in terms of the mean and variance.

4.2 Conception of the Prototype Computer

The first equipment that was proposed for the recording of data for statistical analysis was called a mode-selective bearing error classifier and counter. The proposal was to sample the directional pair voltages at a 10 cycle per second rate, compute the bearing by means of an analog device and record the bearings on a set of selective, electro-mechanical counters. The counter output would consist of data for the cumulative frequency distribution of the bearings, i.e., each counter would record all bearings equal to, or less than, a predeter-

mined angular distance from the great circle bearing of the transmitter. The complete set of readings would represent the histogram of the bearings over the period of the count.

An equipment having the features proposed above was built by Mr. V. E. Peckham and is described in his master's thesis[26]. The equipment operated satisfactorily under laboratory-imposed conditions. It was never tried under field conditions because of two limitations that soon became apparent. The limitations were first, the inability of the circuitry to follow the rapid amplitude fluctuations in the signal, and second, the occurrence of a serious instrumental error in the recorded data due to phase shifts between the computer input voltages. This latter condition gives rise to ellipsing of the bearing as seen on the cathode-ray direction finder.

4.3 Analysis and Reduction of Instrumental Error in the Prototype Computer

The inability of the circuits to follow rapid amplitude fluctuations was circumvented by using direct coupling in all of the electronic computing and processing circuitry. Small fluctuations in the data are still observed which can be traced to the effects of direct coupling. However, this does not result in any serious instrumental error in the modified computer as presently used.

Theoretical study of the data treatment in the prototype computer revealed that a very serious contribution to the instrumental error would occur due to phase shift between the computer inputs. The nature of this error and the means for its reduction is given in Appendix B and the results are summarized in Fig. 36. It is seen that for signals arriving from azimuths that are even integer multiples of 45 degrees, the instrumental error is serious for even small amounts of ellipsing;[27] say less than ten percent. It is also seen that for those signals that arrive from azimuths that are odd integer multiples of 45 degrees, the instrumental error is zero for any amount of ellipsing. Also the error due to ellipsing is not significant for even moderate amounts of ellipsing -- say less than 20 percent, provided the fluctua-

26. Peckham, V.E. *A Bearing Error Data Computer and Counter*, Research Thesis submitted in partial fulfillment of the requirements of a Master's Degree, University of Illinois, Urbana, Illinois, February 1953.

27. By percent of ellipsing is meant one hundred times the ratio of minor to major axis of the ellipse.

tion of the bearing is within ±15 degrees of the most favorable azimuthal angle of arrival. The solution to the error reduction problem is now immediately evident. There is needed a device that can effectively shift the azimuth of the arriving signal to the nearest odd integer multiple of 45 degrees. In practice the majority of the bearings that can be determined have fluctuations well within the ±15 degree range and do not ellipse by more than 10-20%, hence the possession of a bearing shifter would solve the problem.

In connection with the recording of bearing deviations within ±15 degrees of the 45 degree reference, Appendix A demonstrates that the logarithm of the tangent of the angle should be recorded since there is an almost linear correspondence between angle and log tangent of the angle.

4.4 The Design and Construction of the Modified Computer

The block diagram of Fig. 7 depicts the functional processes that are performed by the present model of the analog computer. The two-channel directional information is fed to the bearing shifter which performs the transformations necessary to shift the apparent azimuth of the signal as seen by the computer to the nearest odd integer multiple of 45 degrees. The signal is then sampled at a 25 cps rate, amplified, detected, and fed to a pulse-amplitude stretching circuit. The pulses are then base-clamped at zero by means of a d-c restorer, sent through a logarithmic attenuator, amplified, and their difference taken in a linear difference-bridge circuit. The final output is taken from a low impedance cathode follower amplifier. The output is proportional to the difference of the logarithms of the amplitudes of the directional components and hence is proportional to the quotient of the amplitude of the components. It can be shown that the azimuthal angle of arrival of the signal is just the arc tangent of this quotient. Hence there is available at the output a voltage whose magnitude is proportional to the bearing deviation from a preassigned reference.

The waveform appearing at the output of the computer is not of sufficient amplitude to operate the electro-mechanical counters. A considerable amount of d-c amplification and further pulse processing would be required to properly operate the bearing counters. However, the output voltage is within the range of a DuMont type 279 dual-beam

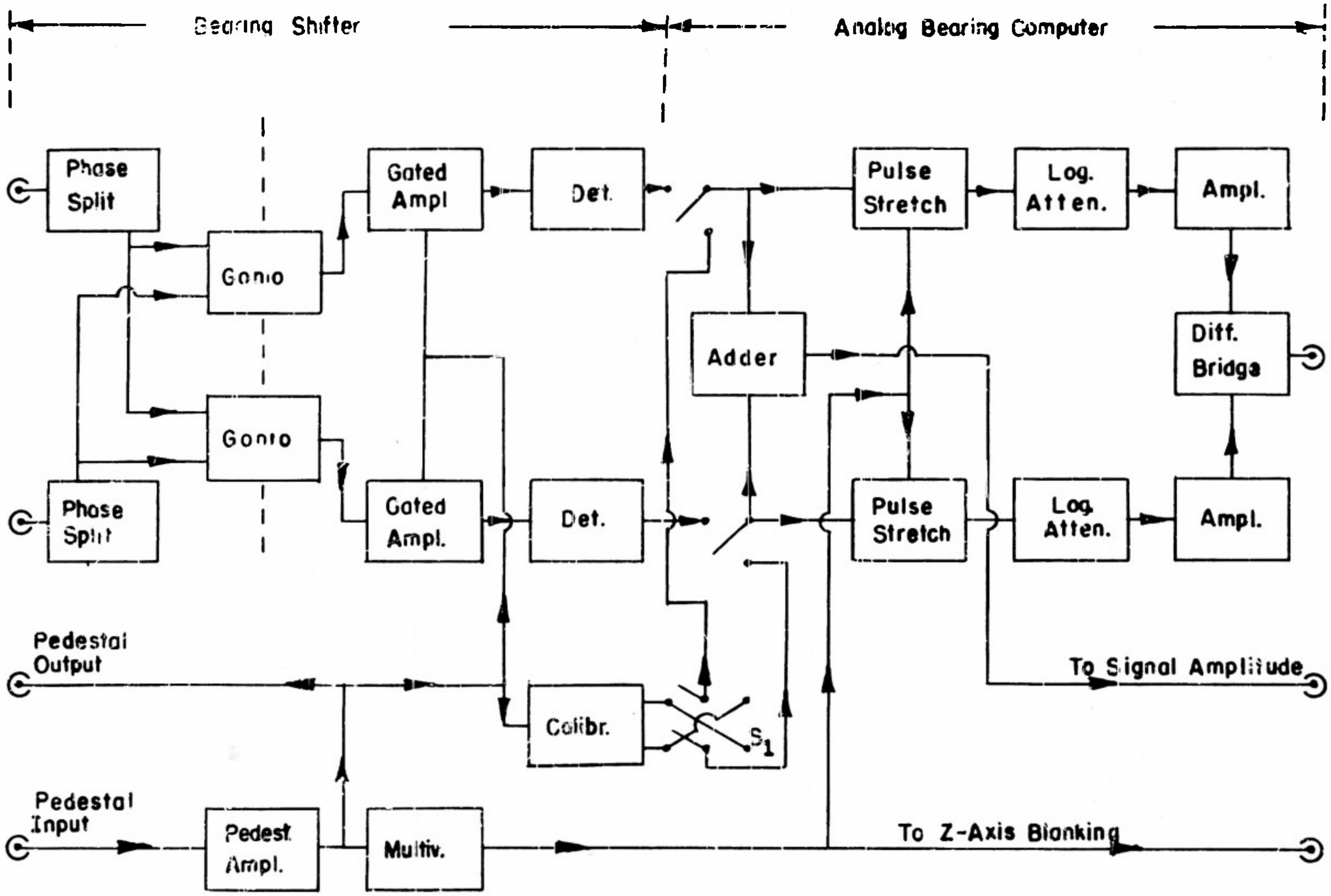


FIGURE 7 BLOCK DIAGRAM OF THE BEARING SHIFTER AND ANALOG BEARING COMPUTER FUNCTIONS.

cathode-ray oscillograph and the bearing display can be conveniently photographed on a continuously moving 35 mm film. A Fairchild Oscillo-Record Camera is an ideal device for this latter purpose. Consequently a compromise was made in the method of data recording in that photographic recording replaced the original intention of digital recording.

There are several advantages to the data recording compromise and at least one disadvantage. The totalizing character of the counters is lost and must be paid for in the routine of visually scanning long strips of 35 mm film. However, the film preserves the time-series sequence of the data and hence permits differentiation between the long-and-short time fluctuations in the bearing deviations. Also correlation studies can be made in the case of photographic records. In addition one can sample at a much greater rate than is possible with the mechanical counters -- the latter being limited to counting rates less than ten per second. It seems necessary that any complete recording equipment should incorporate both the totalizing feature of the counters and the instantaneous cathode-ray tube display. The latter was considered sufficient for the data that were needed in this investigation.

4.5 Reduction of Systematic Errors in the Radio Direction Finder

The two channels of the cathode-ray direction finder were realigned before each recording. However, it is very probable that there may be small residual errors due to unbalances existing in the system after the realignment. Appendix C shows that these small residual errors may be cleared from the recording system at the time the bearing shifter is used to set the zero reference bearing.

4.6 Characteristics of the Analog Computer

An internal calibrator was incorporated in the computer for test purposes. However, the calibration for measurement purposes was made on signals from a target transmitter placed at known azimuths. The bearing shifter was usually adjusted so that the indicated zero reference bearing of the computer coincided with that integer azimuth nearest the great circle bearing of the transmissions being studied. The calculated great circle bearings of the Columbus, Ohio and WWV Beltsville transmissions are 88°47′ and 92°49′ respectively.

The bearing sensitivity of the computer is very good with deviations of less than one-half a degree being readily discernable.

In this respect the computer is better than the cathode-ray direction finder. However, the bearing accuracy of any individual indication is probably not better than one-half degree. This latter is due in part to slight time deviations in the characteristic of the logarithmic attenuator and in part to the shifting of the operating points of the direct coupled amplifiers with change in signal level. The shifting operating point tends to alter slightly the amplifier gain in spite of the negative feed back that is incorporated. On the other hand, the bearing accuracy of the mean of a group of readings taken over a few minutes is considerably less than one-half degree, and comes about because of the averaging process.

The dynamic range of signal amplitudes that can be processed is approximately 24 db. This corresponds to a maximum-to-minimum signal ratio of 15:9 which in turn compares favorably with the dynamic range of the cathode-ray direction finder at any fixed gain. When the bearing ellipse on the cathode-ray direction finder is less than one-sixteenth of its normal size a satisfactory bearing determination cannot be made. Hence any bearing deflection that is readable on the cathode-ray direction finder is within the dynamic range of the computer.

In order to properly assess the indicated bearings it is necessary to know that the signal amplitude is within the dynamic range of the computer. The adder circuit provides a voltage whose amplitude is proportional to signal applied to the computer. This voltage is displayed along with the corresponding bearing data on the screen of the two-beam oscillograph used with the photographic recorder. In addition to the assessment feature this voltage provides a means for correlating bearing error with signal strength.

4.7 Recording of the Bearing and Amplitude Data

A DuMont type 279 dual-beam cathode-ray oscillograph utilizing a P-7 type screen was used for displaying the data The P-7 type screen has a long-persistence yellow phosphor and a short-persistence blue phosphor. A blue-sensitive but otherwise color blind film was used to record the instantaneous bearing and amplitude fluctuations.

The data were recorded on 100 foot rolls of positive film. The Fairchild Oscillo-Record Camera, which was used for the recording, has a continuously variable range of film speeds from one inch per minute

to sixty inches per second. It was found that film speeds of 15 to 25 inches per minute gave recordings that made efficient use of the recording medium yet retained sufficient detail to permit visual analysis of the individual events without the use of projection equipment.

Figure 8 is an example of the recorded data. Each bearing sample is recorded in the main portion of the film as a small, rectangular, white spot that is displaced above (North) or below (South) the reference zero trace. A sample is obtained every one-twenty-fifth of a second and in the particular recording shown, each linear inch of film represents a time interval of approximately three and one-half seconds. The instantaneous signal amplitude is recorded near the lower edge of the film and the amplitude of the envelope is a measure of signal intensity.

An interesting example of a periodic fluctuation in the bearing is noted in the last 30-40 seconds and the bearing fluctuation near the end of the last strip should be compared with Fig. 4. Evidently an example of persistent wave interference error between two signals is being indicated here -- the period of the fluctuation is approximately five seconds. Bearing fluctuation due to polarization would be similar to the symmetrical bearing swing observed near the end of the second strip.

The continuity of the bearing data is good; the isolated "outliers" are usually due to atmospheric noise bursts and are discounted. However, if one were to sample at a rate much less than was used here, say one sample per second, he would lose the continuity of the data and could not generally differentiate so easily between representative bearings and spurious responses. Similarly, little is gained by sampling at a much more rapid rate. One would more completely define the bearing behavior at the times of greatest change; however, this improvement is probably marginal since the definition given by the present sampling rate is sufficient for the majority of cases.

A composite picture of information that is pertinent to the interpretation of the A-scope display is afforded by Fig. 9. Figure 9a is a drawing of the rectified envelope of one complete cycle of the periodic waveform as transmitted from Columbus, Ohio. It consists of a short pulse portion or spike followed by a relatively long, rectangular-shaped pulse portion having sufficient delay to prevent inter-pulse

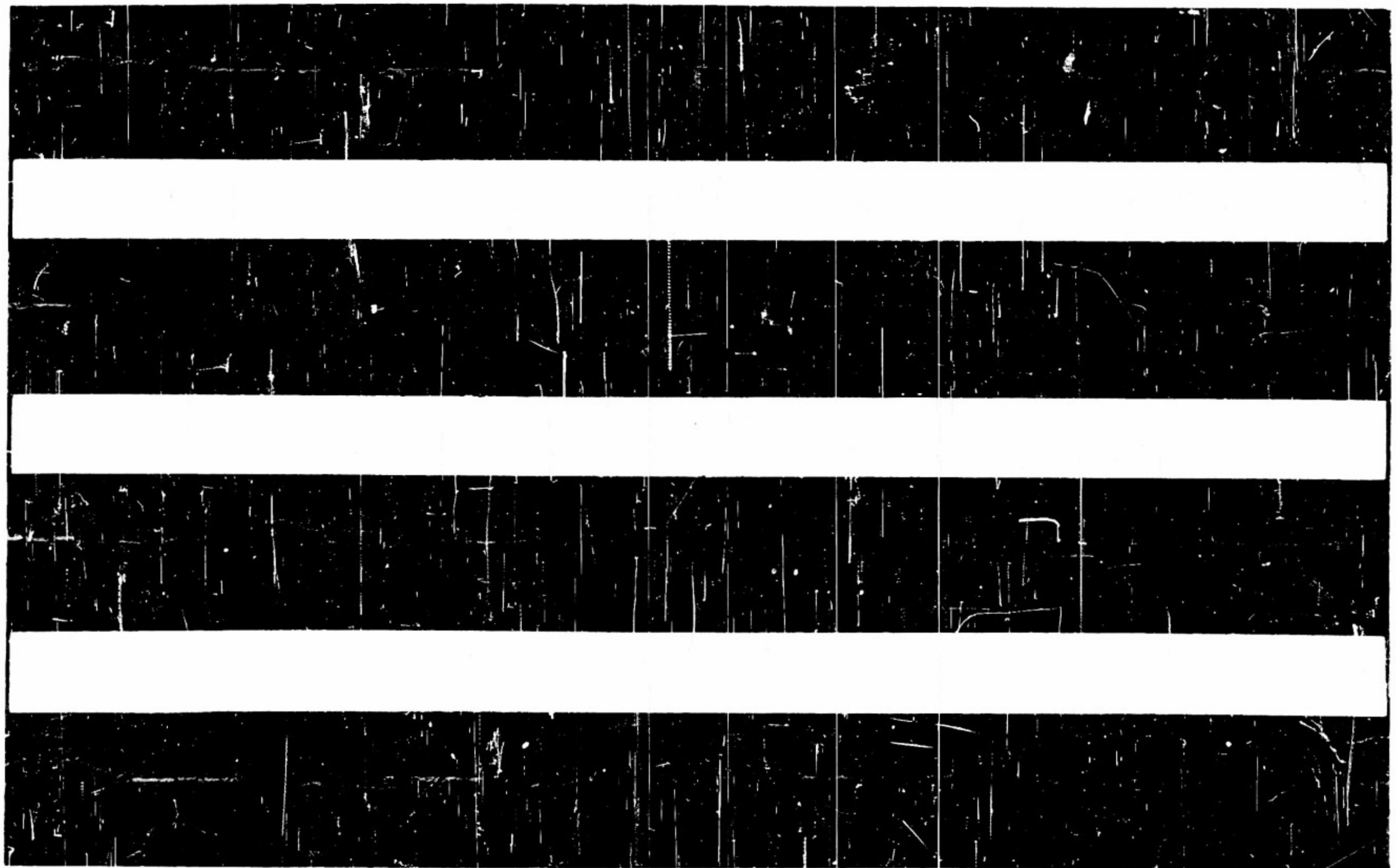
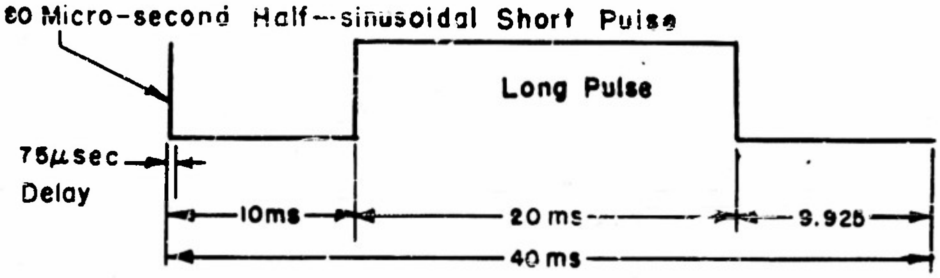
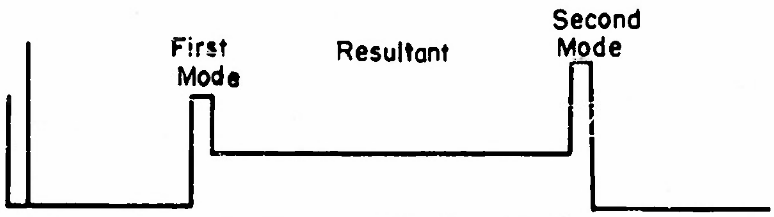


FIGURE 8 BEARING AND SIGNAL AMPLITUDE RECORDINGS -- 4 AUGUST 1953, 3 PM, CDT.
VERTICAL SCALE -- APPROXIMATELY THIRTY DEGREES PER INCH.
HORIZONTAL SCALE -- APPROXIMATELY 3.5 SECONDS PER INCH.
SAMPLING RATE -- TWENTY-FIVE BEARINGS PER SECOND.

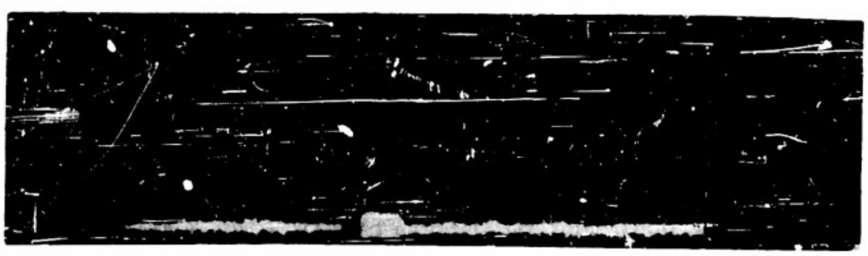


a. Envelope of rectified pulse waveform as transmitted.



b. constructed envelope of rectified pulse waveform as received for the case of two-mode propagation with 1000 micro-sec. delay between modes, relative amplitude ratio of 1.5, and 180° relative phase difference.

c. Photograph of the envelope of rectified pulse waveform as received for the case of two-mode propagation with approximately 1000 micro-sec. delay between modes, approximate relative amplitude of 1.5, and approximately zero degrees of relative phase difference.



d. Photograph of the envelope of rectified pulse waveform as received for the case of two-mode propagation with approximately 1000 micro-sec. delay between modes, approximate relative amplitude ratio of 1.1, and approximately 180° of relative phase difference.

FIGURE 9 PERTINENT TO THE INTERPRETATION OF THE A-SCOPE DISPLAY OF THE RECTIFIED SIGNAL ENVELOPE AS TRANSMITTED AND AS RECEIVED

interference. The particular waveform was chosen because it permits the study of bearing performance of a radio direction finder under very general conditions. Without going into the details it is possible, by the use of such a waveform, to assess the relative performance of a direction finder on instantaneous, coded, or continuous transmissions. The number of transmission modes, relative amplitudes of the signals as received via the several transmission modes, relative time delays between modes, and the relative time phase shifts between interfering modes may also be readily determined.

Figure 9b is a drawing of the rectified envelope of one cycle of the complete waveform as received under a particular ' of wave interference conditions. It should be compared with the actual photographs that appear in the later portion of the figure.

Figures 9c and 9d are actual photographs of two different A-scope displays of received signals. Double mode transmissions are in evidence and the delay between the first and second mode is approximately 1000 microseconds. The two short pulses give information concerning the relative amplitudes of the signals transmitted by the two modes and the relative time delay between modes. The long pulse portion gives this same information and in addition indicates the relative time phase difference between the two modes. The first portion of the long pulse is proportional to the amplitude of the first ray or mode to arrive. Upon the arrival of the second mode, the resultant portion of the long pulse may either increase or decrease depending upon the relative time phase difference between the two modes. After the termination of the first mode, only the second mode appears and this accounts for the abrupt increase in amplitude near the end of the long pulse portion. The pass band of the receiver was not sufficient to accommodate the significant spectrum of the short pulses. Consequently the pulse amplitudes are smaller than their long-pulse counterparts.[28]

Continuous visual monitoring of the A-scope display was made during the course of each bearing recording to observe the propagation conditions that were in evidence and to maintain the relative position of the bearing sampling gate with respect to the propagation mode being studied.

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28. Radio direction finders of the narrow base type inherently have low sensitivity and hence must be designed to have very limited bandwidth in order that the signal-to-noise ratio be tolerable. The limited bandwidth permits greater gain per stage of amplification by virtue of the constant gain band-width product principle, but unfortunately for this case, the concomitant frequency and delay distortion must be accepted.

5. THE REDUCED DATA

5.1 Method of Data Reduction

5.1.1 Data Recovery

The indicated bearing and signal amplitude data were recorded on 35 millimeter film. In order to recover the data in a form that is convenient for statistical treatment a read-out device similar to a conventional film editor was constructed. A strip of millimeter cross-sectional paper was pasted on the base to provide a graduated background for the transparent film. The data were read out in terms of millimeter squares and the observer was instructed to estimate the data to the nearest half square. The conversion from squares to degrees was made by carefully measuring the bearing deviation calibration portion of each film strip and using this information for the conversion. In the majority of cases, two degrees of bearing deviation corresponded to 3/50 of an inch or 3/2 of a millimeter.

5.1.2 Sampling Procedure

The procedure for sampling was affected by first scanning an entire film to find those sections that exhibited the greatest bearing fluctuation. Samples were then taken from these sections -- the assumption being that the data so obtained would be indicative of direction finder behavior under wave interference and/or polarization conditions. The majority of the samples were of approximately five minutes duration. However, occasions would arise where a burst of inter-station interference would obfuscate part of the data and consequently a shorter length sample had to be accepted.

A single sample consisted of the mean of approximately 10 successive indicated bearings, i.e., the indicated bearing averaged over four-tenths of a second. Thirty-five consecutive samples were averaged together to determine one point of data for plotting purposes.

5.1.3 Types of Reduced Data

Corresponding to each point of bearing data a mean value, a progressive or cumulative mean value, and a standard deviation were calculated. The mean range of the bearing was studied for only a

few periods since it did not yield the significance that is usually attributed to it -- the reason for this will be discussed later. A correlation coefficient between signal amplitude and bearing error was calculated for one period of sampling. The results did not appear useful enough the justify the additional expense of calculating a correlation coefficient for each study. A censored bearing study was made for a few periods. The criterion for censoring was to select "strong and steady bearings". In executing the censoring, the observer was instructed to select bearings for those occasions where (1) the signal was greater than one-half its peak value, and (2) the rate of change of bearing was zero. As in the case of the correlation study, there was a noticeable improvement in the bearing accuracy that one could obtain but the improvement was not as significant as that due to averaging over all bearings.

5.1.4 Definition of Pertinent Symbols

The symbols that are defined below apply to the plotted curves of reduced data.

$\bar{B}_i$ is the mean value of thirty-five consecutive samples of indicated bearing.

$\bar{B}_p$ is the progressive or cumulative mean for successive large sample means ($N = 35$) taken from the start of the period up to and including the point in question. This is the quantity that is of most interest in this investigation.

$\bar{B}_c$ is the censored mean value of those samples in a consecutive set of thirty-five that corresponds to the "strong and steady" bearing criterion.

σ is the standard deviation of a set of thirty-five successive samples.

$\bar{R}$ is the mean range of thirty-five consecutive samples.

$\bar{R}/\sigma$ is the ratio of the mean range to the standard deviation. For samples of size 10 drawn from a normally distributed population the mean range-to-sigma ratio should be 3.08.

A is the amplitude of the envelope of the vertical component of the received signal.

5.1.5 Pertinent Statistical Formulas

For each point of data the mean was calculated from the formula

$$\bar{B}_i = \frac{\Sigma_1^n B_i}{n} \text{ where } 0 < i \leq 35.$$

The standard deviation was calculated from the formula

$$\sigma = \sqrt{\frac{\Sigma_1^n (B_i)^2}{n} - \frac{(\Sigma_1^n B_i)^2}{n(n-1)}}$$

The correlation coefficient was calculated as

$$\frac{\text{Expectation of } (|B|\ A)}{\sqrt{(\text{Variance of } |B|)\ (\text{Variance of } A)}}$$

where the expectation of $(|B|\ A)$ was calculated as

$$\frac{\Sigma_1^n |B_i|\ A_i}{n} - \bar{A}\ |\bar{B}_i|$$

5.1.6 Removal of Residual Systematic Error

For each film a systematic error had to be estimated and from this the estimated great circle bearing line was entered on the curves of reduced data for the pertinent film. The estimation consisted in correcting for (1) the reference zero setting of the computer, (2) observer error in the readout, and (3) a systematic error of approximately 40 minutes due to site calibration. There was a strong temptation (which was resisted) to let the mean of the cumulative means determine the great circle bearing because this value appears to be very stable over the periods that were studied. Considerably more data would be needed to establish the correctness of such a practice, although the central-limit theorem of statistics implies the trends that were observed.

The reduced data are plotted in the section that follows. The cumulative mean is probably the most interesting of the several characteristic curves because this value tends toward the true (great circle bearing) mean except for lateral deviation effects.

5.2 Curves of Reduced Data

The bearing recording studies were initiated on 28 July 1953. The first film was exposed for purposes of gaining experience in calibrating the film and finding the optimum recording speed, oscillograph beam intensities, and camera aperture. Data-wise the film was considered a "dry run". The bearings were subject to very little swinging and/or fading at this time, hence were uninteresting from the viewpoint of wave-interference and/or polarization effects on the direction of arrival of radio waves. No reduction was made of the data recorded on film one.

Fourteen additional bearing recordings were made and certain statistics have been calculated and plotted as the curves of reduced data that follow. The data reduction represents approximately 500 manhours of read-out, calculation, and plotting.

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER TWO, SHOWN IN FIG. 10

Date.--30 July 1953

Film roll number.--Two

Approximate Time.--3:05 to 3:25 p.m. C.D.T.

Number of bearings per sample.--9

Number of bearings per point of data.--315

Elapsed time per point of data.--12.6 seconds

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulsed waveform having 25 cps wave repetition frequency

Pertinent remarks from field notebook and log.--

(1) Evidence of ordinary and extraordinary ray propagation was exhibited during the course of the twenty-minute recording period.

(2) Strong signals for the most part -- ellipsing of the bearing coincided with amplitude fading.

Remarks pertinent to the data reduction.--

A. First study is for conditions prevailing at approximately 3:05 p.m.

B. Second study is for conditions prevailing at approximately 3:20 p.m.

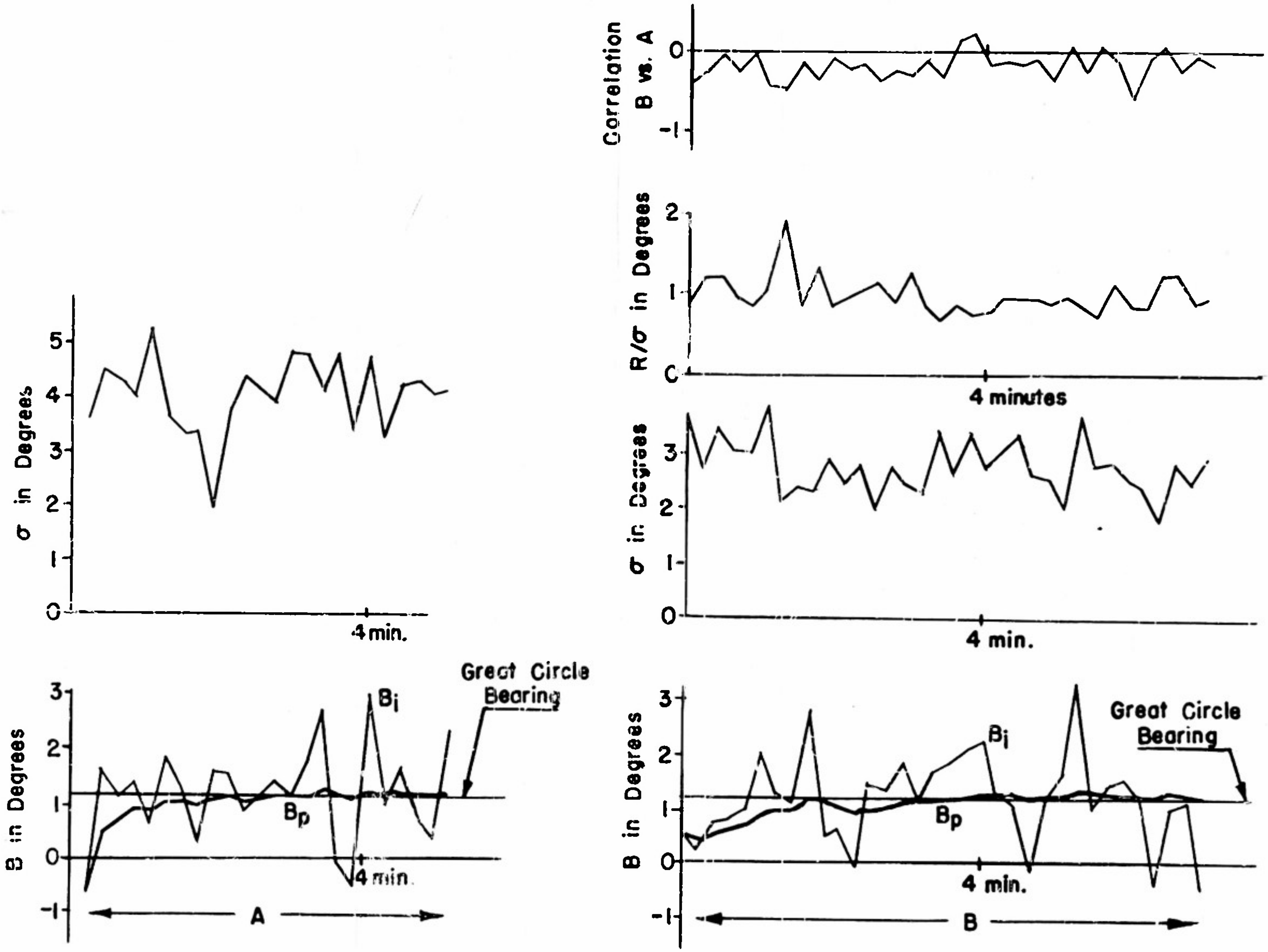

FIGURE 10 PERTINENT TO BEARING RECORDING NUMBER TWO

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER THREE, SHOWN IN FIG. 11

Date.--31 July 1953

Film roll number.--Three

Approximate time.--Ten - twelve a.m. C.D.T.

Number of bearings per sample.--9

Number of bearings per point of data.--315

Elapsed time per point of data.--12.6 seconds

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulsed waveform having a 25 cps waveform repetition rate.

Pertinent remarks from field notebook and log.--

(1) A ten-minute recording was made of bearing conditions prevailing at the beginning of the period, 10:00 to 10:15 a.m. A second ten-minute recording was made of bearing conditions prevailing at the end of the period 11:45 to 11:55 a.m.

(2) Ordinary and extraordinary ray propagations were in evidence at the beginning of the period with consequent ellipsing of the bearing and fading of the signal.

(3) Bearing deviation was limited to approximately ± one degree at end of period with little evidence of multipath propagation.

Remarks pertinent to the reduced data.--

A. 5.3-minute study from the period 10:00 to 10:15 a.m.

B. 6.7-minute study from the period 11:45 to 11:55 a.m.

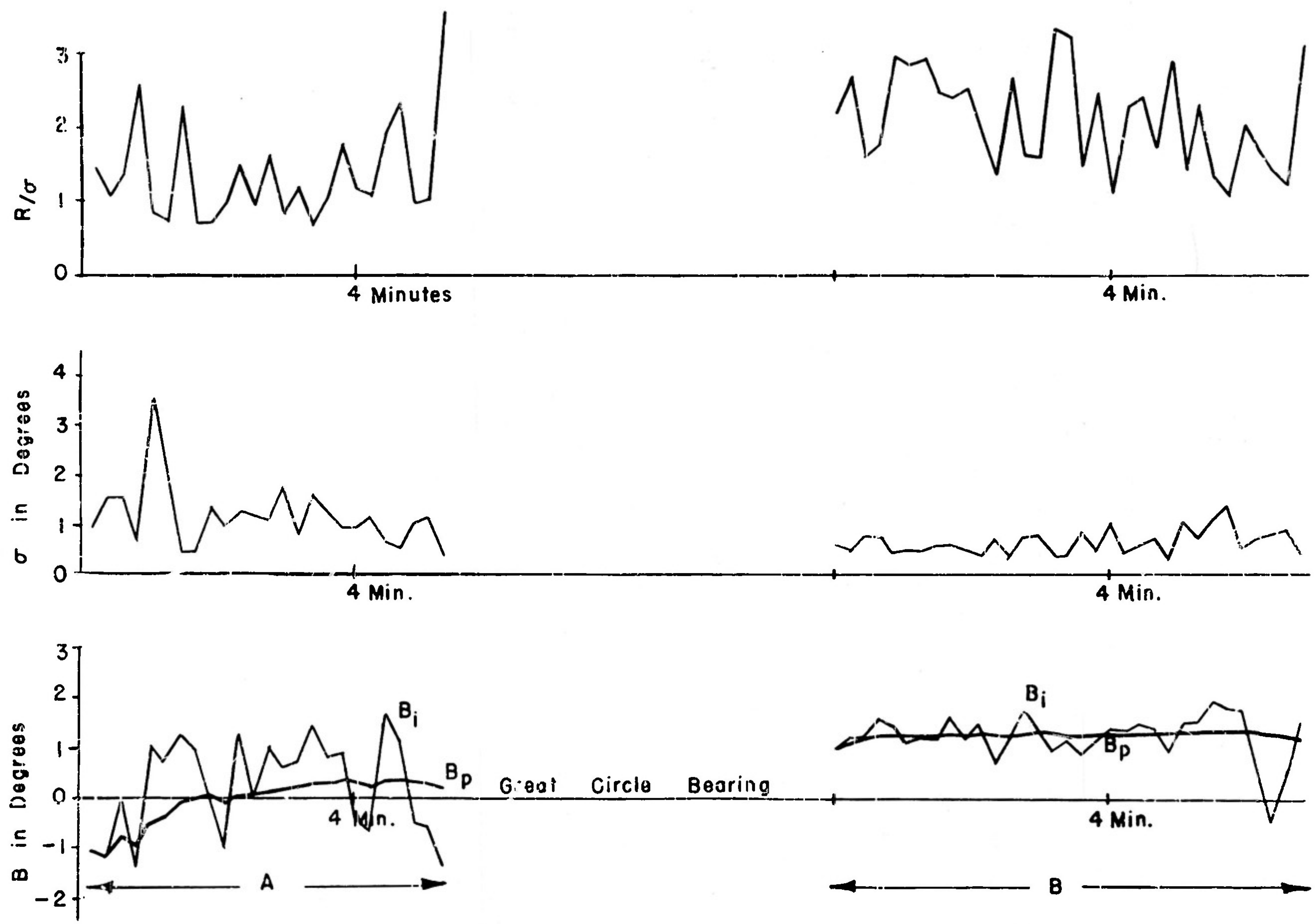

FIGURE 11 PERTINENT TO BEARING RECORDING NUMBER THREE

SUMMARY OF INFORMATION PERTINENT TO
THE DATA REDUCTION OF BEARING RECORDING
NUMBER FOUR, SHOWN IN FIG. 12

Date.--3 August 1953

Film roll number.--Four

Approximate time.--3 to 4 p.m. C.D.T.

Number of bearings per sample.--9

Number of bearings per point of data.--315

Elapsed time per point of data.--12.6 seconds

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmissions.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps.

Modulation.--Special pulsed waveform having a 25 cps waveform repetition frequency.

Pertinent remarks from the field notebook and log.--

(1) The first half of the film recorded bearing conditions prevailing during the period of 3:00 to 3:15 p.m., and the second half of the film recorded the bearing conditions during the period 3:45 to 4:00 p.m.

(2) The indicated bearing exhibited continuous fluctuation throughout period with occasional ellipsing of the bearing and with concurrent amplitude fading. Multipath propagation effects were in evidence at the end of the period.

Remarks pertinent to the reduced data.--

A. Four minute study for bearing conditions prevailing at approximately 3:05 p.m.

B. Four minute study of bearing conditions prevailing at approximately 3:50 p.m.

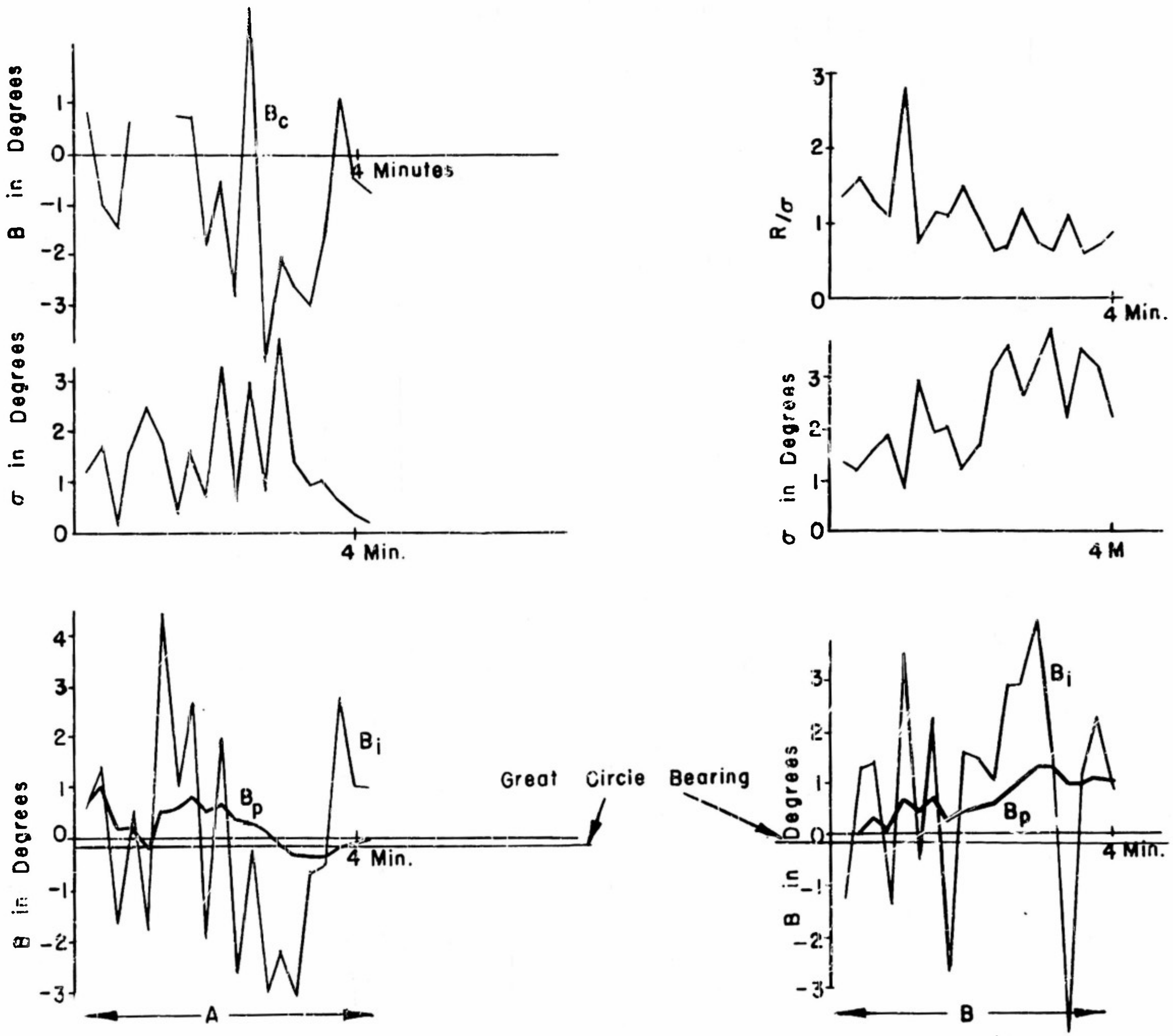

FIGURE 12 PERTINENT TO BEARING RECORDING NUMBER FOUR

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER FIVE, SHOWN IN FIG. 13

Date.--4 August 1953

Film roll number.--Five

Approximate time.--Three to four p.m. C.D.T.

Number of bearings per sample.--30

Number of bearings per point of data.--1050

Elapsed time per point of data.--42 seconds

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulsed waveform having a 25 cps waveform repetition frequency.

Pertinent remarks from the field notebook and log.--

> Two distinct modes of propagation were in evidence at the beginning of the period with approximately 1200 microseconds of delay between modes. After fifteen minutes the transmission exhibited only an occasional trace of multipath. At the end of the period the signal was weakening and the noise background level was increasing.

Remarks pertinent to the reduced data.--

A. First study for 6.3 minute period after the beginning of the recording.

B. Second study for a 5.6 minute period beginning at approximately 3:15 p.m. C.D.T.

C. Third study for a 7.0 minute period just prior to the end of the recording.

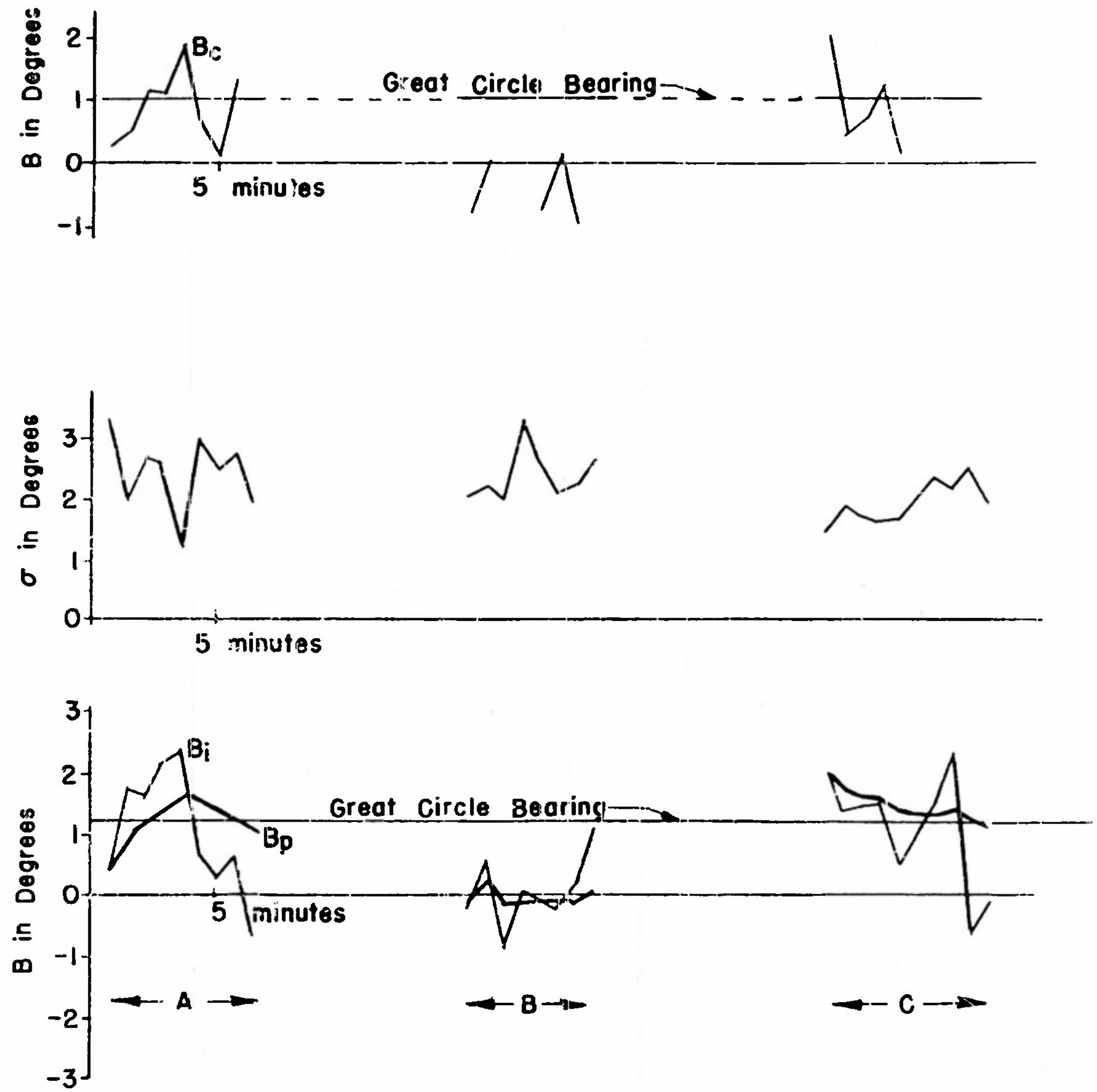

FIGURE 13 PERTINENT TO BEARING RECORDING NUMBER FIVE

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER SIX, SHOWN IN FIG. 14

Date.--6 August 1953

Film roll number.--Six

Approximate time.--10:00 - 11:00 a.m. C.D.T.

Number of bearings per sample.--10

Bearings per point of data.--350

Elapsed time per point of data.--14 seconds

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulsed waveform having 25 cps wave repetition frequency

Pertinent remarks from the field notebook and log.--

(1) Two mode multipath propagation was in evidence with swinging and ellipsing of the bearing and fading of the signal amplitude. Approximately five hundred micro-second delay was observed between the modes.

(2) Three distinct modes occurred later with ±10 degrees or more of bearing fluctuation.

(3) At end of transmission the later modes were still in evidence but with reduced intensity.

Remarks pertinent to the data reduction.--

A. First sample commencing at approximately 10:00 a.m.

B. Second sample commencing at approximately 10:10 a.m.

C. Third sample commencing at approximately 10:30 a.m.

D. Fourth sample commencing at approximately 11:40 a.m. with Triple-mode multipath effects in evidence.

E. Fifth sample commencing at approximately 10:50 a.m.

F. Sixth sample commencing at approximately 11:00 a.m.

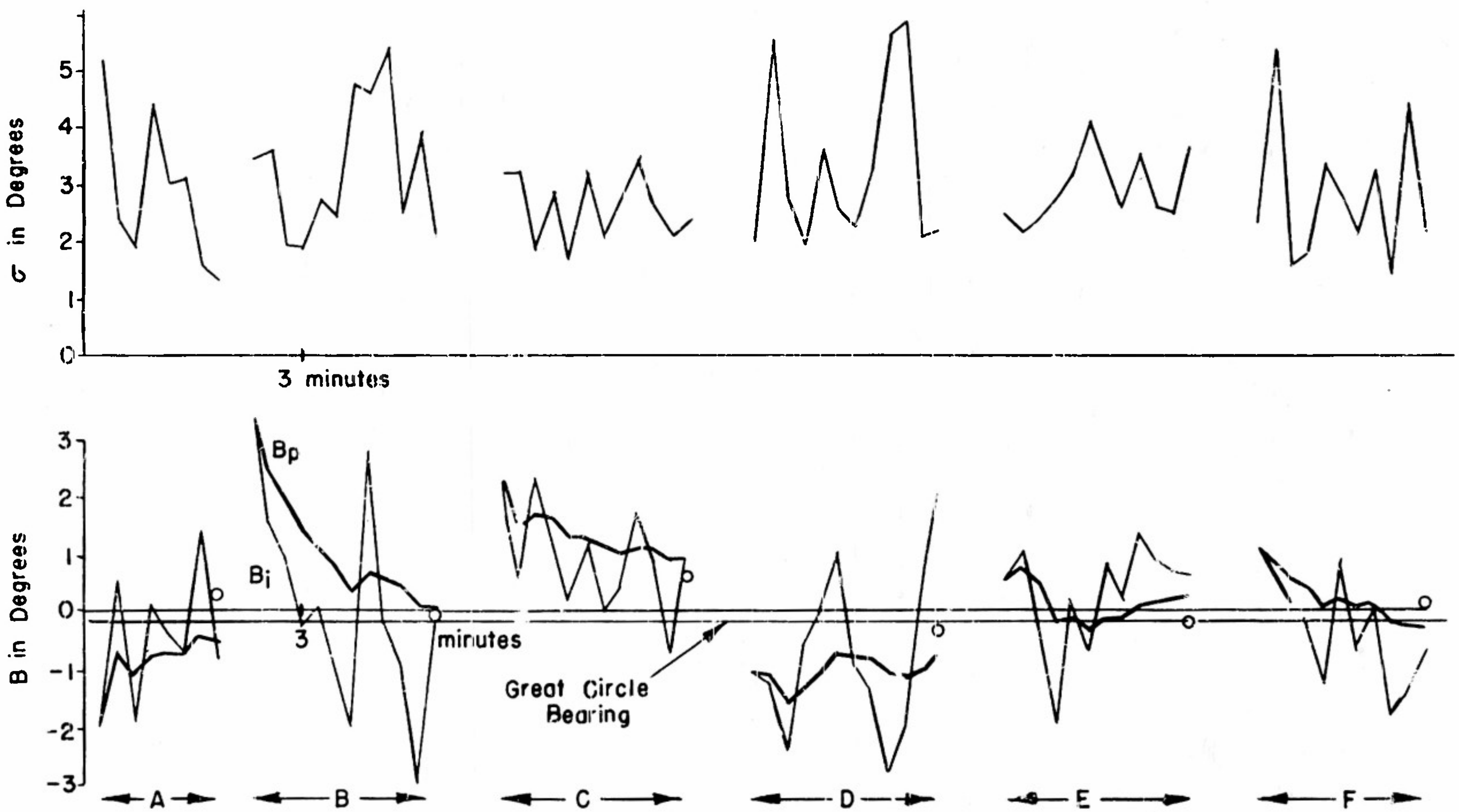

FIGURE 14 PERTINENT TO BEARING RECORDING NUMBER SIX

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER SEVEN, SHOWN IN FIG. 15

Date.--6 August 1953

Film roll number.--Seven

Approximate time.--3:00 - 4:00 p.m. C.D.T.

Number of bearings per sample.--11

Number of bearings per point of data.--385

Elapsed time per point of data.--15.4 seconds

Bearing deviation calibration of film.--2° - 4/50 inch

Source of Transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 cps

Modulation.--Special pulsed waveform having 25 cps wave repetition frequency.

Pertinent remarks from field notebook and log.--

(1) At start of period the bearing fluctuation was very slow with a pronounced lateral deviation to the south and with single mode propagation in evidence.

(2) There was a general increase in noise and in bearing fluctuation, swinging, and ellipsing as the hour passed.

(3) At the end of transmission there was evidence of ordinary and extraordinary ray propagation with large bearing swings.

(4) This should be an interesting case to analyze.

Remarks pertinent to the data reduction.

A. First study commencing at approximately 3:00 p.m. exhibited little swinging but pronounced lateral deviation to the south.

B. Second study commencing at approximately 3:10 p.m. exhibited little swinging an no evidence of lateral deviation.

C. Third study commencing at approximately 3:20 p.m. exhibited single mode transmission with noise level coming up and bearing fluctuation increasing.

D. Fourth study commencing at approximately 3:35 p.m. exhibited ellipsing of the bearing with greater fluctuation of the bearing.

E. Fifth study commencing at approximately 3:45 p.m. showed evidence of multipath error, probably due to ordinary and extraordinary ray interference.

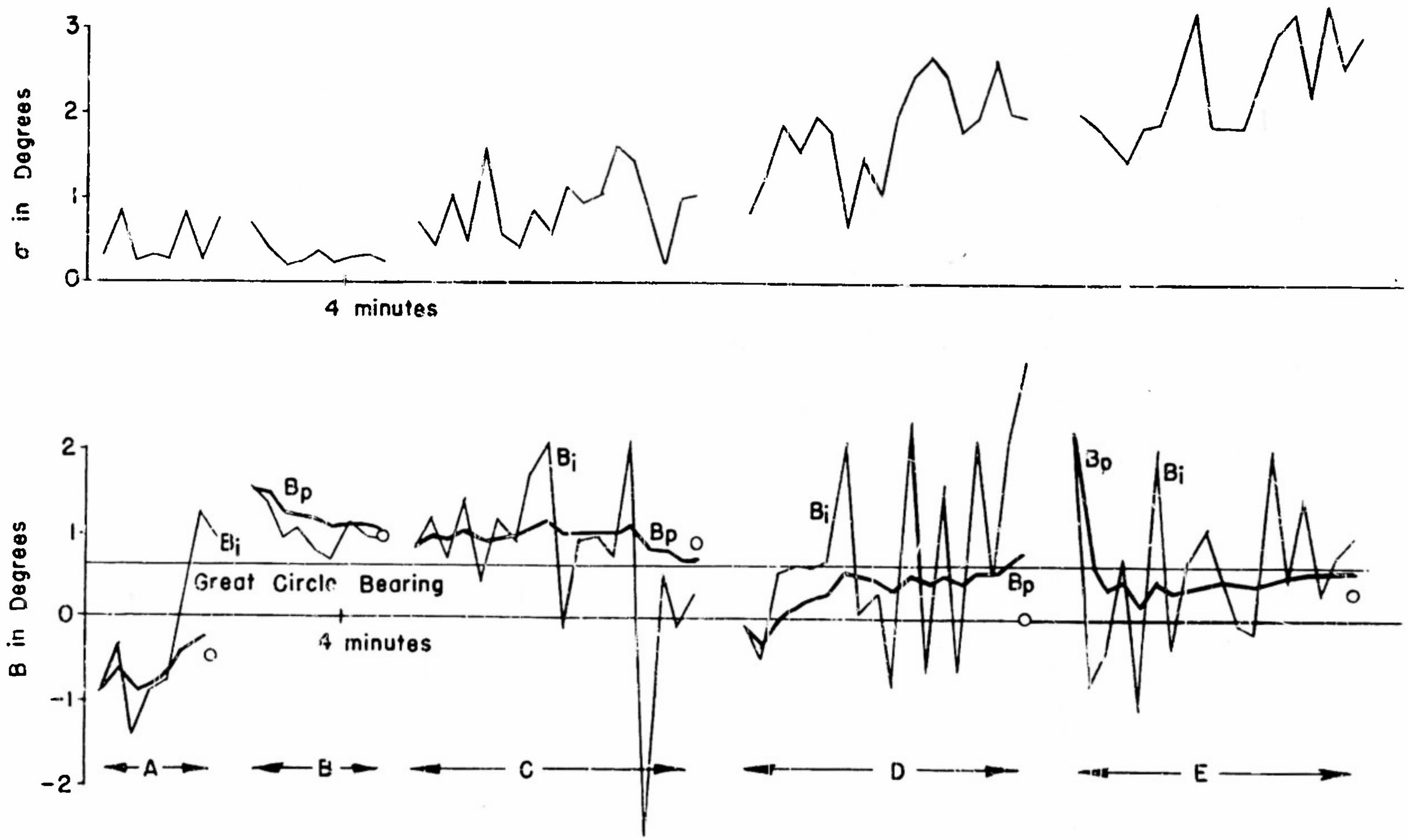

FIGURE 15 PERTINENT TO BEARING RECORDING NUMBER SEVEN

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER EIGHT, SHOWN IN FIG. 16

Date.--7 August 1953

Film roll number.--eight

Approximate time.--10:00 - 11:00 a.m. C.D.T.

Number of bearings per sample.--11

Number of bearings per point of data.--385

Elapsed time per point of data.--15.4 seconds

Bearing deviation calibration of the film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulse waveform having 25 cps waveform repetition frequency

Pertinent remarks from the field notebook and log.--

(1) At beginning of period multipath transmissions via two distinct modes separated approximately 500 microseconds in time were in evidence; the second mode had approximately one-half the amplitude of the first mode. Occasional extraordinary and ordinary ray propagations were in evidence.

(2) By 10:20 a.m. only occasional traces of second mode were noticed and bearing fluctuation had decreased.

Remarks pertinent to the data reduction.--

A. First study commencing at approximately 10:05 a.m. during time of multipath transmission.

B. Second study commencing at approximately 10:25 a.m. during time of predominant single mode transmission.

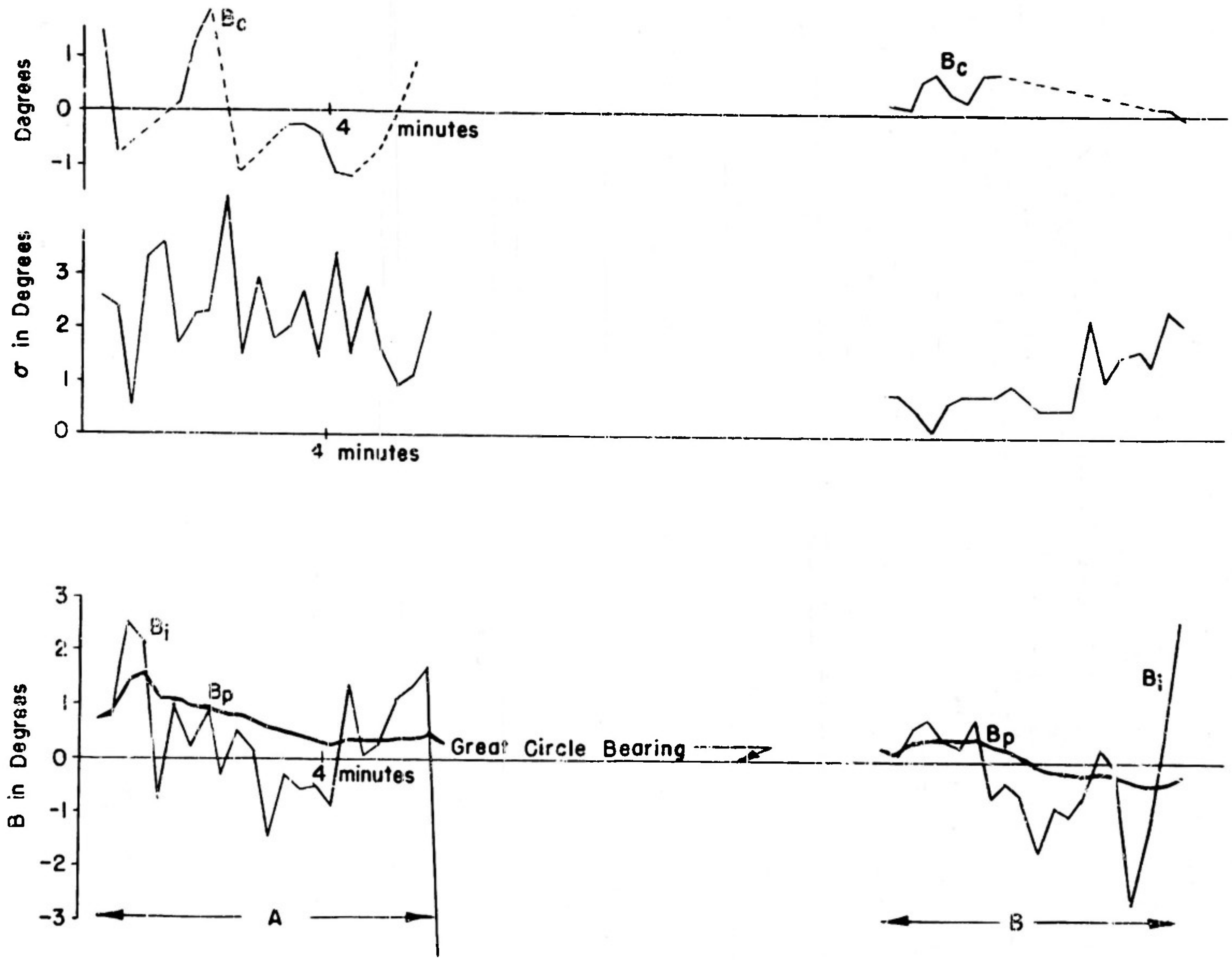

FIGURE 16 PERTINENT TO BEARING RECORDING NUMBER EIGHT

SUMMARY OF INFORMATION PERTINENT TO THE
DATA REDUCTION OF BEARING RECORDING
NUMBER NINE, SHOWN IN FIG. 17

Date.--7 August 1953

Film roll number.--Nine

Approximate time.--3 - 4 p.m. C.D.T.

Number of bearings per sample.--10

Number of bearings per point of data.--350

Elapsed time per point of data.--14 seconds

Bearing deviation calibration film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulse waveform having 25 cps waveform repetition frequency.

Pertinent remarks from field notebook and log.--

A. First study commencing at 3:00 p.m. showed evidence of lateral deviation to South with slow bearing fluctuation but no visible evidence of multipath transmission.

B. Second study commencing at approximately 3:15 p.m. showed no evidence of lateral deviation.

C. Third study commencing at approximately 3:30 p.m. showed little change from previous condition.

D. Fourth study shows the effect of increasing noise level on the fluctation of the bearing.

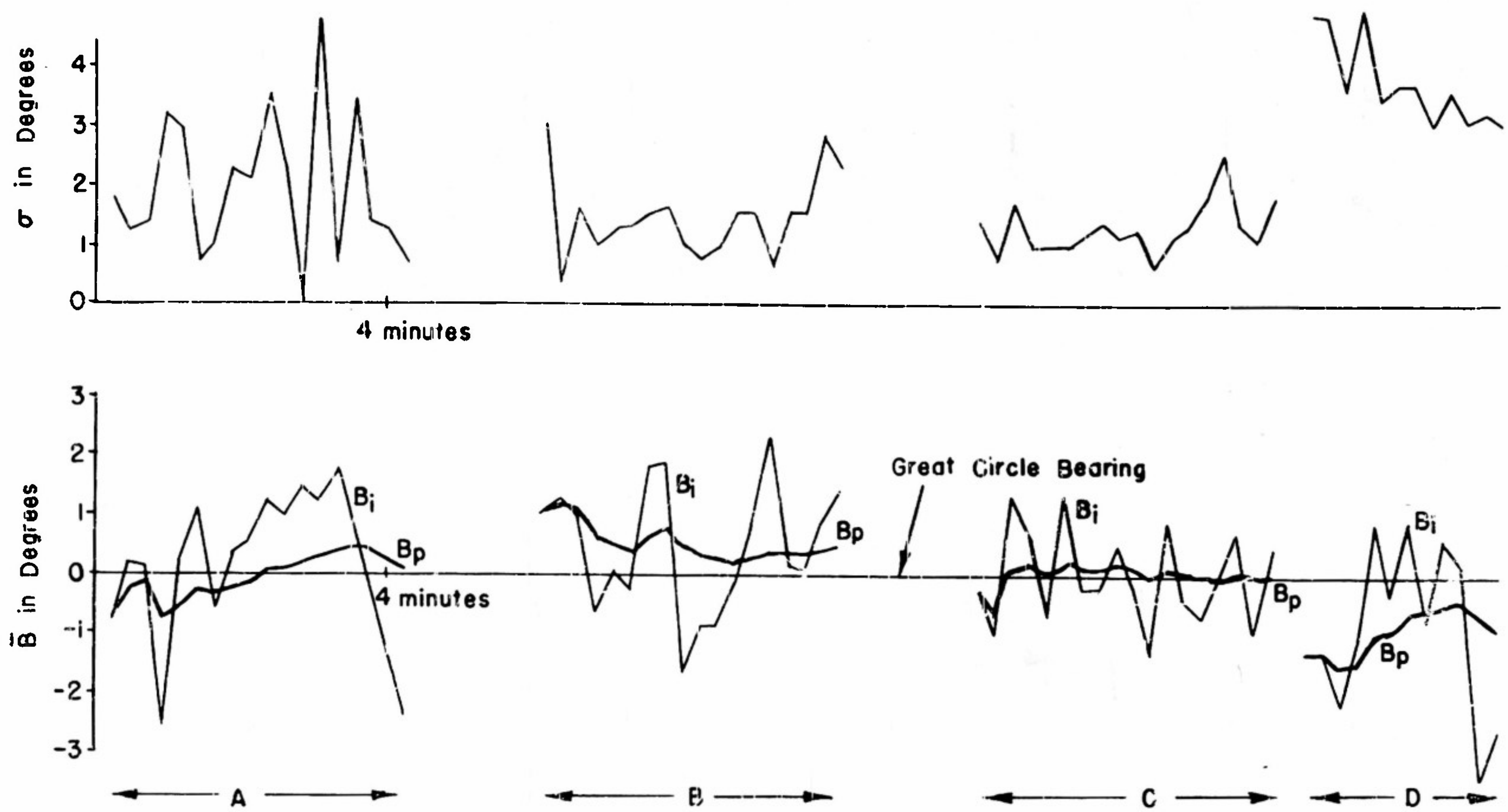

FIGURE 17 PERTINENT TO BEARING RECORDING NUMBER NINE

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER TEN, SHOWN IN FIG. 18

Date.--27 October 1953

Film roll number.--Ten

Approximate time.--2 - 3 p.m. C.S.T.

Number of bearings per sample.--10

Number of bearings per point of data.--350

Elapsed time per point of data.--14

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulse waveform having 25 cps waveform repetition frequency

Pertinent remarks from the field notebook and log.--

(1) The mode selective feature of recorder was made use of in this study since two and three mode propagations were in evidence throughout the complete period of study. The delay between successive modes was approximately 500 microseconds.

Remarks pertinent to the data reduction.

A. Single mode conditions prevailing at approximately 2:00 p.m.

B. Single mode conditions prevailing at approximately 2:06 p.m.

C. Conditions prevailing at approximately 2:09 p.m. due to interference between first two modes.

D. Conditions prevailing at approximately 2:17 p.m. due to interference between first two modes.

E. Conditions prevailing at approximately 2:20 p.m. due to single mode.

F. Conditions prevailing at approximately 2:33 p.m. due to wave interference between two modes.

G. Conditions prevailing at approximately 2:37 p.m. due to single mode; lateral deviation to north in evidence.

H. Conditions prevailing at approximately 2:40 p.m. due to wave interference between first two modes.

I. Conditions prevailing at approximately 2:42 p.m. due to wave interference between first two modes.

J. Conditions prevailing at approximately 2:43 p.m. due to wave interference between first three modes.

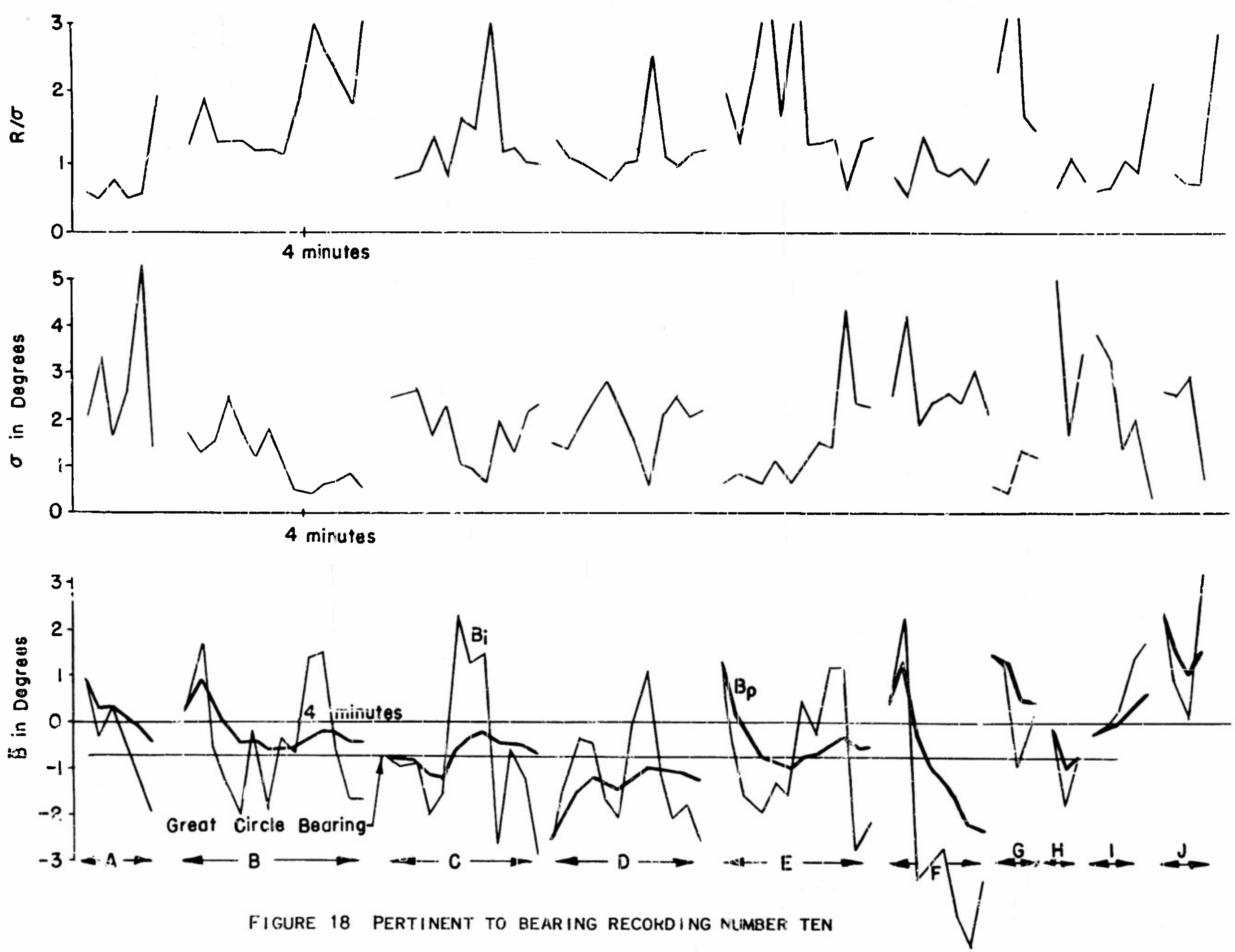

FIGURE 18 PERTINENT TO BEARING RECORDING NUMBER TEN

SUMMARY OF INFORMATION PERTINENT TO
THE DATA REDUCTION OF BEARING RECORDING
NUMBER ELEVEN, SHOWN IN FIG. 19

Date.--28 October 1953

Film roll number.--Eleven

Approximate time --2 - 3 p.m. C.S.T.

Number of bearings per sample.--10

Number of bearings per point of data.--350

Elapsed time per point of data.--14 seconds

Bearing deviation calibration of film.--2° = 1/25 inch

Source of transmission.--WWV, Beltsville, Maryland

Carrier frequency.- 5000 kcps

Modulation.--400 cps tone periodically replaced by voice modulation.

Pertinent to the data reduction.--

A. Bearing condition prevailing at approximately 2:00 p.m.

B. Bearing condition prevailing at approximately 2:10 p.m.

C. Bearing condition prevailing at approximately 2:20 p.m.

D. Bearing condition prevailing at approximately 2:25 p.m.

E. Bearing condition prevailing at approximately 2:30 p.m.

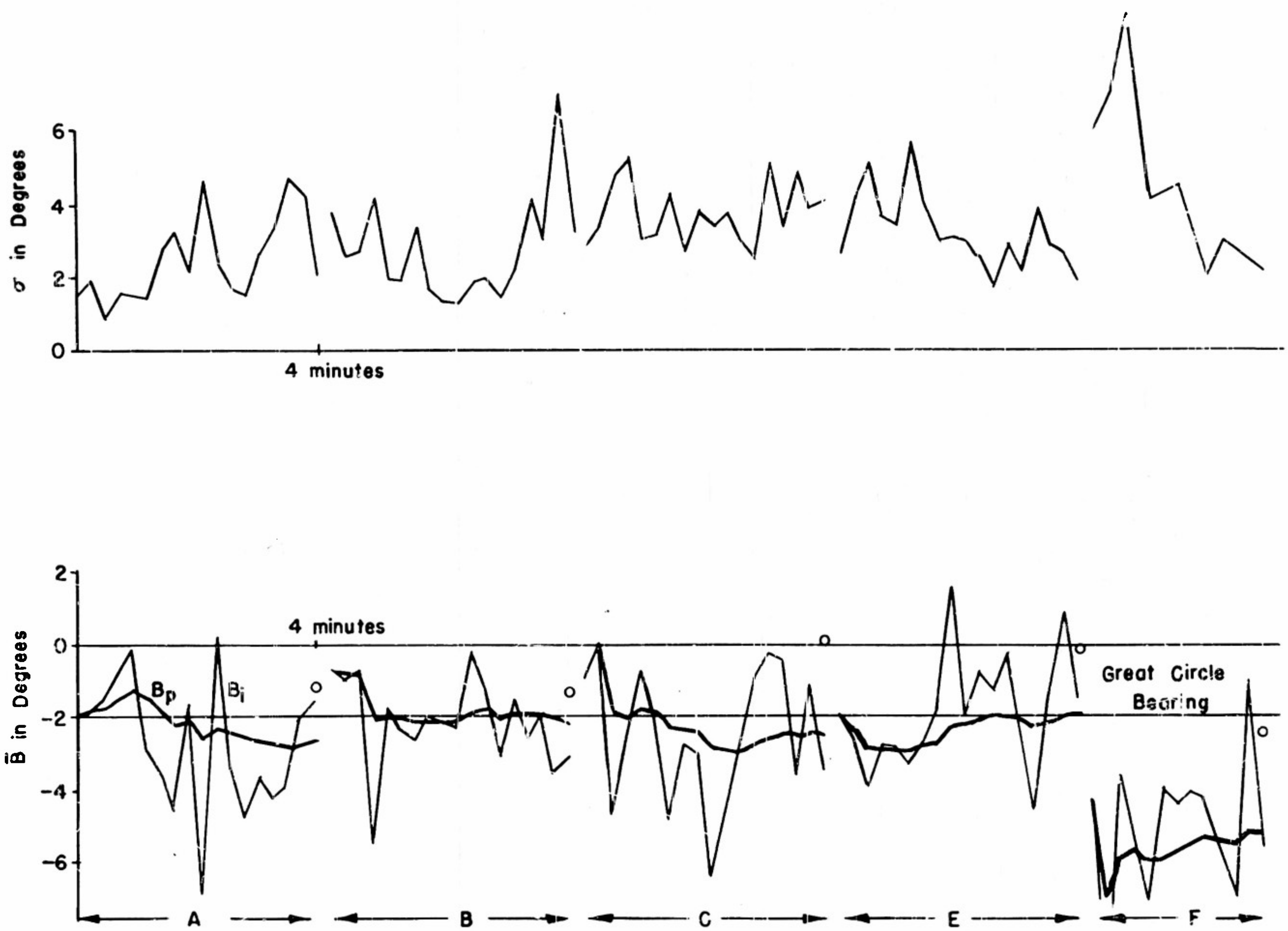

FIGURE 19 PERTINENT TO THE BEARING RECORDING NUMBERED ELEVEN

SUMMARY OF INFORMATION PERTINENT TO
THE DATA REDUCTION OF BEARING RECORDING
NUMBER TWELVE, SHOWN IN FIG. 20

Date.--29 October 1953

Film roll number.--Twelve

Approximate time.--2 - 3 p.m. C.S.T.

Number of Bearings per sample.--10

Number of Bearings per point of data.--350

Elapsed time per point of data.--14

Bearing deviation calibration of film.--2° = 3/50 inch

Source of transmission.--Ohio State University, Columbus, Ohio

Carrier frequency.--6420 kcps

Modulation.--Special pulse waveform having 25 cps waveform repetition frequency

Pertinent remarks from the field notebook and log.--

The mode selective feature of recorder was made use of in this study since two and three mode propagations were in evidence throughout the complete period of study. The delay between successive modes was approximately 500 microseconds.

Remarks pertinent to the data reduction.--

A. Conditions prevailing at approximately 2:00 p.m. due to interference between first and second modes.

B. Conditions prevailing at approximately 2:04 p.m. due to first mode alone.

C. Conditions prevailing at approximately 2:12 p.m. due to interference between first and second modes.

D. Conditions prevailing at approximately 2:18 p.m. due to first mode alone.

E. Conditions prevailing at approximately 2:26 p.m. due to interference between first and second modes. Lateral deviation to south in evidence.

F. Conditions prevailing at approximately 2:34 p.m. due to first mode alone. Increased noise level resulted in more rapid fluctuation of the bearing and greater deviation.

G. Conditions prevailing at approximately 2:37 p.m. due to interference between first and second modes. Increased noise level resulted in more rapid fluctuation of the bearing and greater deviation.

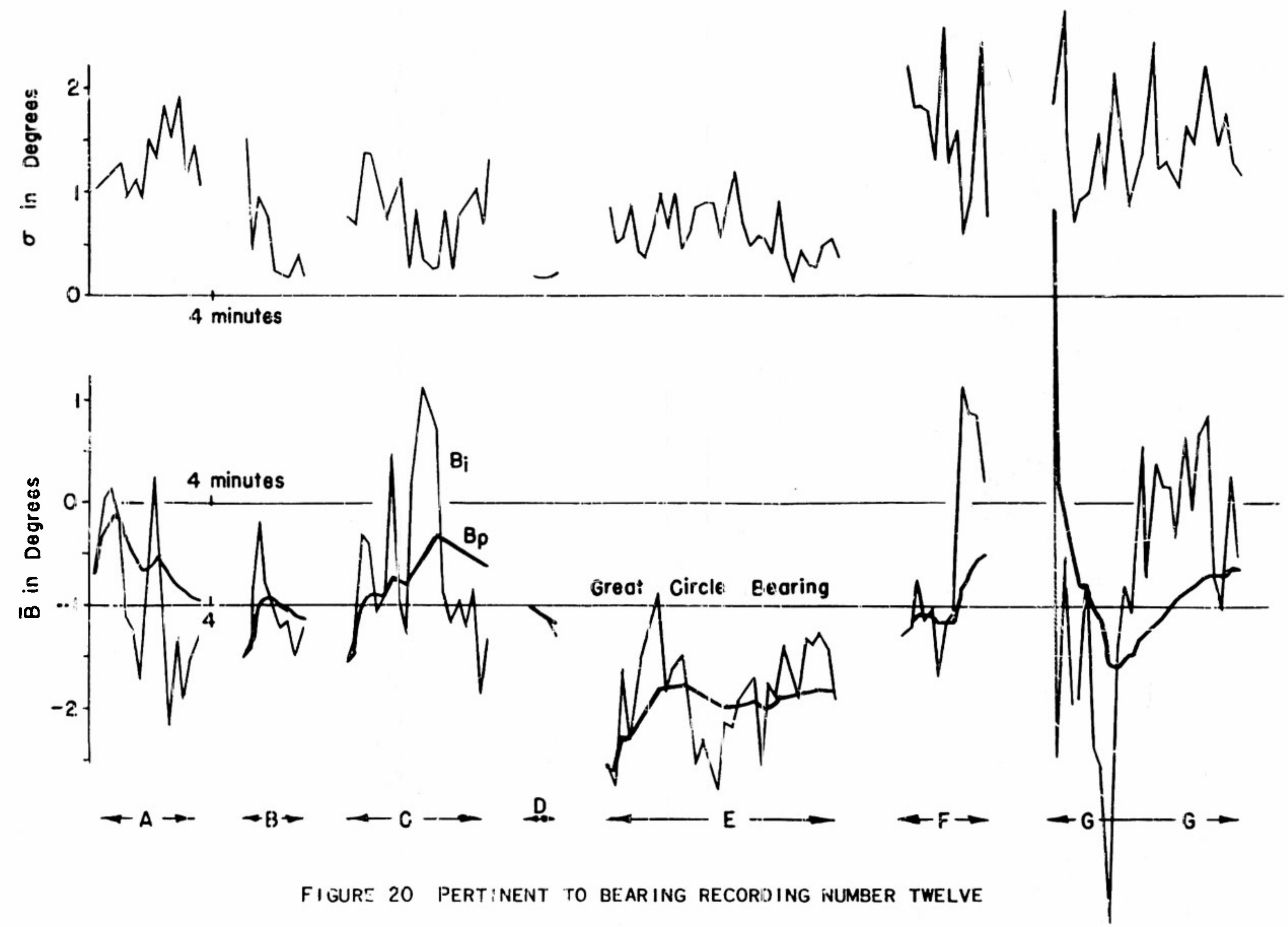

FIGURE 20 PERTINENT TO BEARING RECORDING NUMBER TWELVE

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER THIRTEEN, SHOWN IN FIG. 21

Date.--12 February 1954

Film roll number.--Thirteen

Approximate time.--10:48 - 11:25 a.m. C.S.T.

Number of bearings per sample.--8

Number of bearings per point of data.--280

Elapsed time per point of data.--11.2 seconds

Bearing deviation calibration of film.--2° = 1/25 inch

Source of transmission.--WWV, Beltsville, Maryland

Carrier frequency.--5000 kcps

Modulation.--400 cps tone periodically displaced by voice modulation

Pertinent to the data reduction.--

A. Four and one-half minute sample of bearing condition prevailing at approximately 10:50 a.m.

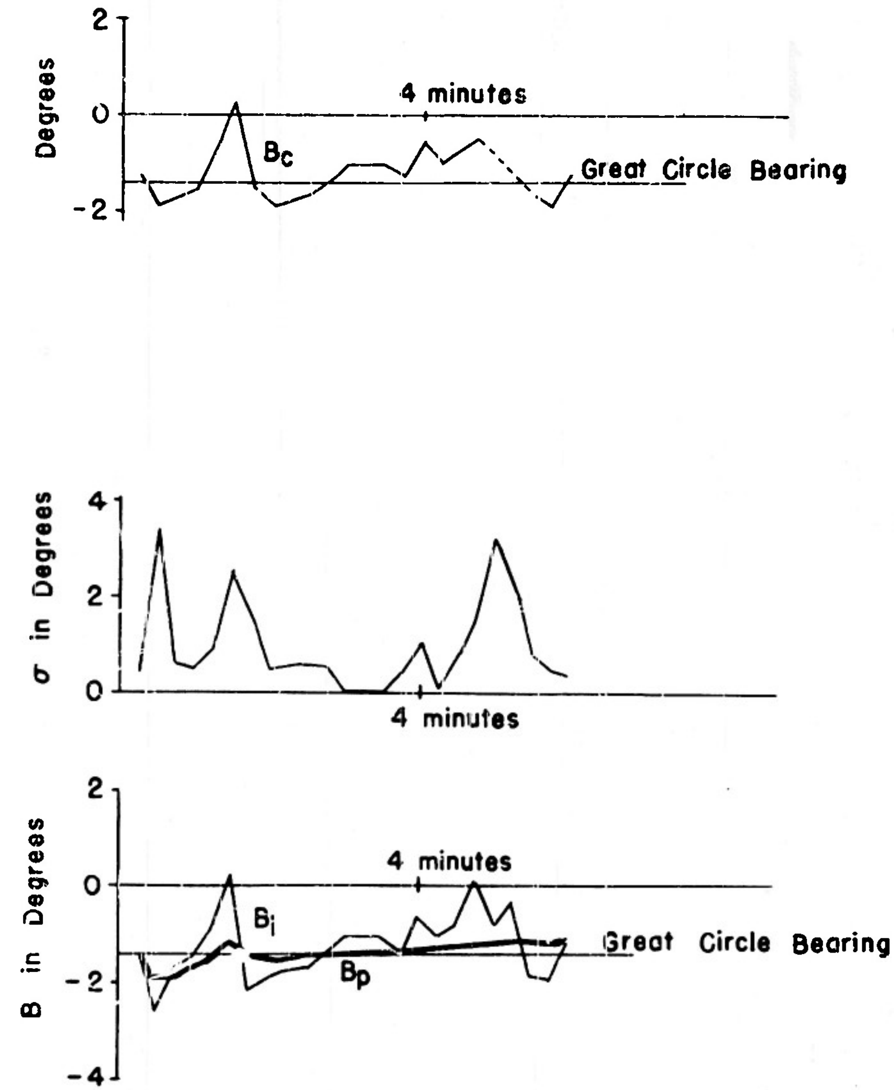

FIGURE 21 PERTINENT TO BEARING RECORDING NUMBER THIRTEEN

SUMMARY OF INFORMATION PERTINENT TO THE DATA REDUCTION OF BEARING RECORDING NUMBER FOURTEEN, SHOWN IN FIG. 22

Date.--12 February 1954

Film roll number.--Fourteen

Approximate time.--3:45 to 4:30 p.m. C.S.T.

Number of bearings per sample.--9

Number of bearings per point of data.--315

Elapsed time per point of data.--12.6 seconds

Bearing deviation calibration of film.--2° = 1/25 inch

Source of transmission.--WWV, Beltsville, Maryland

Carrier frequency.--5000 kcps

Modulation.--400 cps tone periodically displaced by voice modulation

Pertinent to the data reduction.--

A. Five minute sample of bearing conditions prevailing at approximately 3:50 p.m.

B. Five minute sample of bearing conditions prevailing at 4:15 p.m.

C. Five minute sample of bearing conditions prevailing at approximately 4:20 p.m.

D. Five minute sample of bearing conditions prevailing at approximately 4:25 p.m.

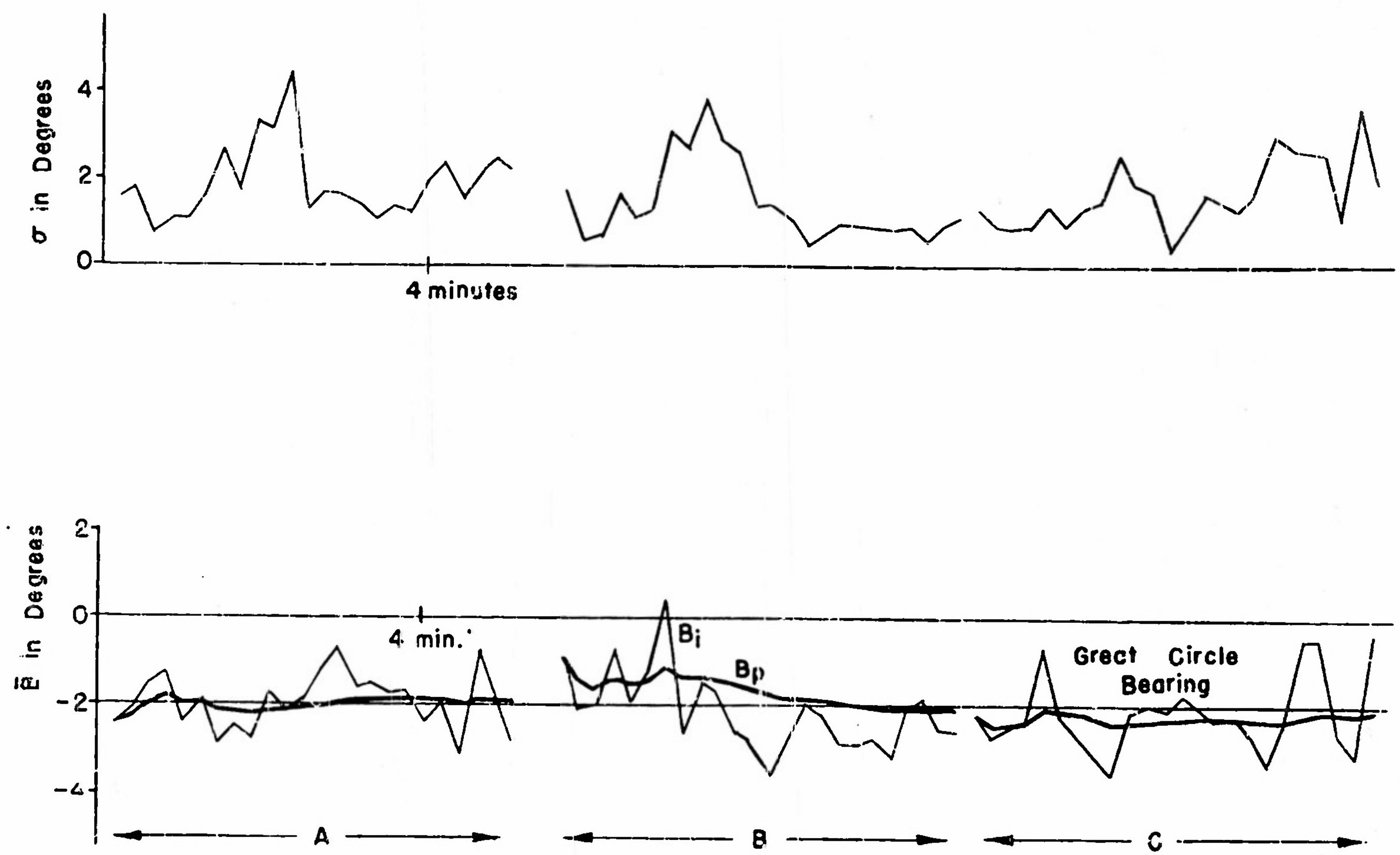

FIGURE 22 PERTINENT TO BEARING RECORDING NUMBER FOURTEEN

SUMMARY OF INFORMATION PERTINENT TO
THE DATA REDUCTION OF BEARING RECORDING
NUMBER FIFTEEN, SHOWN IN FIG. 23

Date.--12 February 1954

Film roll number.--Fifteen

Approximate time.--8:05 to 8:40 p.m. C.S.T.

Number of bearings per sample.--9

Number of bearings per point of data.--315

Elapsed time per point of data.--12.6 seconds

Bearing deviation calibration of film.--2° = 1/25 inch

Source of transmission.--WWV, Beltsville, Maryland

Carrier frequency.--5000 kcps

Modulation.--400 cps tone periodically displaced by voice modulation

Pertinent to the data reduction.--

A. Five minute samples of bearing conditions prevailing at approximately 8:10 p.m.

B. Five minute sample of bearing conditions prevailing at 8:30 p.m.

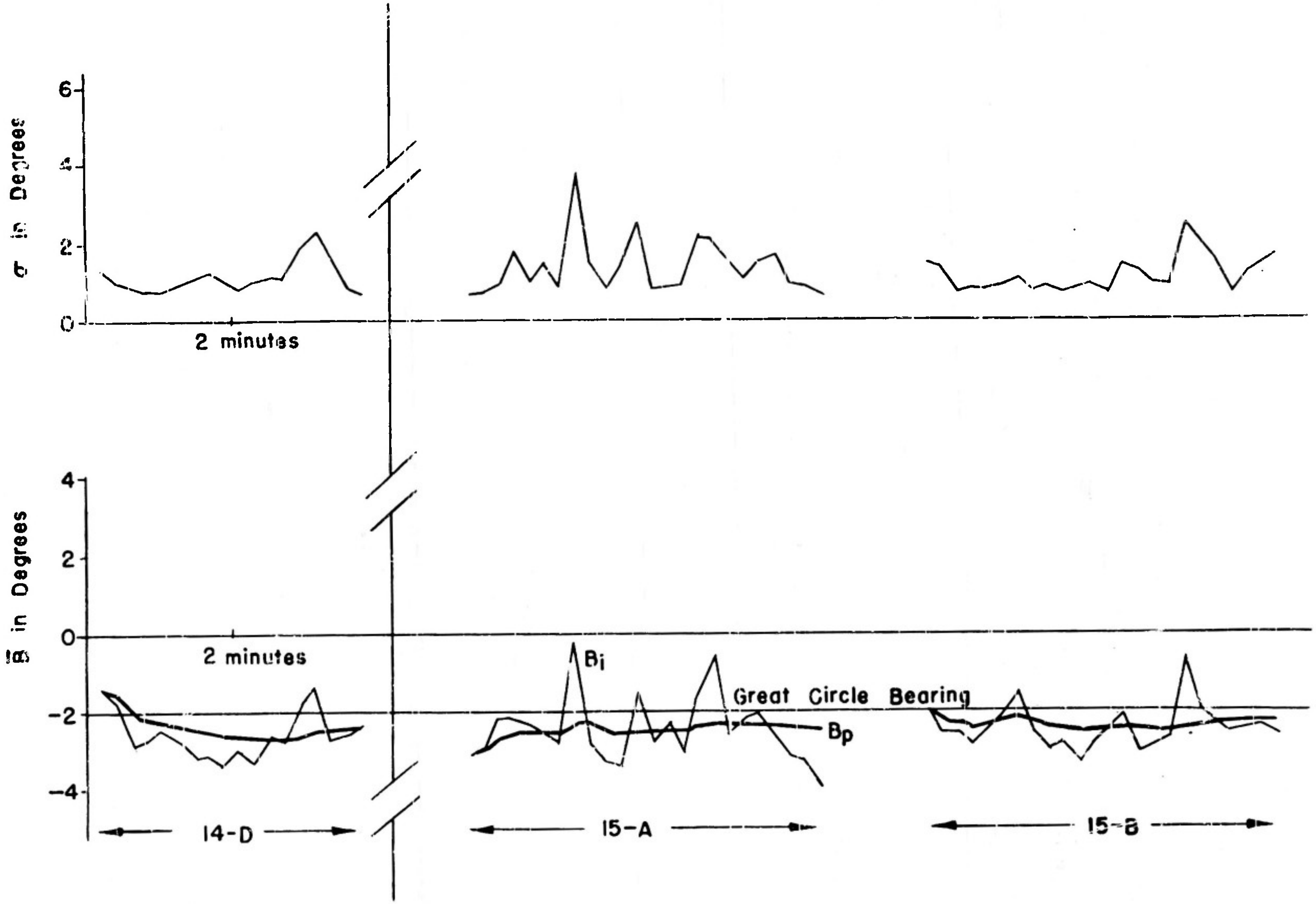

FIGURE 23 PERTINENT TO BEARING RECORDING NUMBER FIFTEEN

6. ANALYSIS OF RESULTS

6.1 Discussion

In the summary evaluation of a large amount of data that is subject to statistical analysis, one selects those statistics which appear to be most useful to the purpose at hand. The data that are shown in Section 5 were subject to several statistical analyses. Those statistics that showed most promise of yielding pertinent information were studied in considerably greater detail than those that indicated little significance. The discussion will be brief and general; where it is necessary to refer to a particular period of a particular study the bearing recording number and associated upper case letter will be given in that order. As an example, 9-B refers to the second sampling period in bearing recording number nine.

6.1.1 Mean and Standard Deviation

Inspection of the indicated bearing mean, $\bar{B}_i$, and the standard deviation of the indicated bearings, σ, for any and all bearing recordings and periods studied reveals that the behavior is typical of small base radio direction finders. The indicated bearing is in a state of constant fluctuation, and the amount of fluctuation is measured by the standard deviation. In general the indicated bearing fluctuates about the great circle bearing with a standard deviation that changes from period to period and from day to day. Bearing recording 7 is a particularly interesting example of the behavior of the indicated mean bearing under conditions of changing standard deviation. It happened that during the time of the recording there was a gradual increase in the noise level and other wave interference effects over the course of the observation period. The steady increase in the standard deviation of the indicated bearing from approximately one-half degree at the outset to approximately 3 degrees at the end of the recording is immediately evident as is the corresponding fluctuation of the indicated bearing mean itself. An even more interesting aspect of this particular study (neglecting a pronounced lateral deviation effect at the beginning) is the stability of the progressive or cumulative mean of the indicated bearing mean. In each of the five cases it is observed that this value tends rather steadily to a value that is very close to the estimated great circle bearing.

The stability of the cumulative mean is demonstrated in practically every case that was studied. The few exceptions can be attributed to lateral deviation effects for the most part. What is being exhibited here is an experimental verification of the implications of the central limit theorem of statistics. This is considered to be one of the important observations that can be drawn from the experimental data.

6.1.2 The Mean Range

The mean range has been used to estimate the standard deviation. In fact W. Ross suggests its use in connection with the statistical treatment of bearings. For samples of size 9 to 10 as used here, the ratio of the mean range to the standard deviation should be approximately 3 or slightly larger.[29] However, very few of the cases studied exhibit this ratio--the mean value is more nearly unity. Pertinent studies are shown by 2-B, 3-A, 3-B, 4-A, 4-B, and 10-A through 10-J. In looking for the reason for the great discrepancy it was found that consecutive samples taken over a period of 0.4 second each would not permit the range to exhibit its true extent. Hence the evaluation of range to sigma ratio was discontinued after a few studies.

6.1.3 Correlation Studies

To learn the extent of the improvement in accuracy of the determination of the direction of arrival of radio waves that one can expect from using a single criterion for the selection of "good" bearings, a correlation study was made between signal amplitude and indicated mean bearing. The only recorded study was that of 2-B. Inspection of the results shows that the mean correlation coefficient is approximately 0.2. If the bearing error were due solely to simple wave interference phenomena, this coefficient should approach -1.0 as a limit. Evidently there are other causes that are acting. Schemes that propose to improve the accuracy of a radio direction finder by censoring bearings according to the large amplitude signal criterion should show some improvement, but the expense of the additional equipment is probably not justified if the coefficient of correlation is generally as low as was the case here.

A set of criteria that may be used is that of selecting only those bearings that are strong and steady. This implies that the amplitude of the signal be large and that the rate of change of bearing deviation

29. F. E. Grubbs and C. L. Weaver, "The Best Unbiased Estimate of Population Standard Deviation Based on Group Ranges," *Journal American Statistical Association*, Vol. 47, 1947, pp. 224-241.

be zero. These criteria were applied in several instances -- see $\bar{B}_c$ in 4-A, 5-A, 5-B, 5-C, 8-A, 8-B, and 13-A. In all of these instances it was noticed that the censored mean fluctuated in a manner not too different from that of the indicated mean although the extent of the fluctuation was reduced. One loses the continuity of the indicated bearing mean by such devices and this may be psychologically undesirable in many instances. If one takes a cumulative average of the censored mean value, he obtains an estimate that is reasonably close to the true value as can be inferred from the above instances. This fact is also illustrated by the *encircled* points in 6-A to 6-F, 7-A to 7-F, and 11-A to 11-F. This procedure is probably most representative of what is done in present day direction finding. However, it appears from these experimental results that the bearing that one obtains in this manner is not as good as could be obtained by complete averaging of all of the indicated bearings.

6.1.4 Mode Selection

Bearing recordings numbered 10 and 12 were carried out under conditions resembling ray selection techniques. The latter recording is more representative of what may be expected to happen. Cases 12-B, and 12-D are for single mode propagation. In each case the indicated mean bearing fluctuation is not extensive and is centered on the great circle bearing. Case 12-F is an example of single mode bearing behavior when atmospheric noise is an additional error causing agent. Cases 12-A, 12-C, 12-E and 12-G are examples of two mode propagation and hence show the effects of wave interference on the mean and standard deviation of the indicated bearing. Attention is again invited to the stability of the cumulative mean. Except for the case of lateral deviation in 12-E, the time trend of this later mean is toward the great circle bearing.

6.1.5 Effect of Distance

Bearing recordings numbered 11, 13, 14, and 15 are based upon transmissions from WWV Beltsville, Maryland. All other recordings are based upon transmissions from Columbus, Ohio. The former station is approximately 1100 kilometers distant while the latter is approximately 450 kilometers. The more distant transmissions appear to

have greater stability of the cumulative mean and this observation is not an unusual one because, as was shown in Section 2.2, lateral deviation effects decrease with increasing distance. However, the number of cases studied for the longer distance transmission path is probably not great enough to warrant making any conclusive statements in this respect.

6.1.6 Effect of Propagation Conditions

The bearing recordings numbered one to nine inclusive were obtained at times of the day and year for which the occurrence of E and F ionospheric-layer reflections were most probable. This was done in order to maximize the chances for the occurrence of wave interference effects in the direction of arrival measurements. The Central Radio Propagation Laboratory predictions were used for establishing the times of most probable occurrence. The laboratory also records quality figures for radio propagation and geomagnetic activity. It is instructive to add that the majority of the bearing recordings were obtained at times when the radio propagation conditions were described as "fair to good."[30] This bore out what was qualitatively observed on the cathode-ray direction finder, i.e., the results that were obtained were representative of what had been observed to happen on the average.

The bearing recordings numbered ten to fifteen inclusive were obtained at times of the year when the occurrence of F layer reflections alone were most probable. However, in terms of the cumulative mean bearing there is no significant difference from the earlier studies.

6.2 Inference

Because the cumulative mean of the indicated bearing mean exhibited a stability about the great circle bearing that was not approached by any other statistic considered here, it was selected as the best characteristic number that can be used in determining the direction of arrival of radio waves by means of a small-base direction finder system. An interval of time is necessary in which the cumulative mean may be evaluated. To obtain a specific answer to the question. "How long does it take to get a good bearing?" histograms of the cumulative mean of the indicated bearing mean for all studies were plotted for

30. U. S. Department of Commerce, National Bureau of Standards, *Radio Propagation Quality Figures for July, August and October, 1953*.

intervals of 0.4 second, approximately 14 seconds, one minute, two minutes, three minutes, and four minutes. The histograms are shown in Figs. 24 and 25. The steady reduction in dispersion with time is again in evidence. To summarize all of this information into one compact set of curves, the cumulative frequency distributions have been drawn on probability paper and the smoothed results are shown in Figs. 26-30. The implication as to a proper method for getting a "good bearing" is now complete. One must first qualify what he considers to be a "good bearing" in terms of a standard deviation; the time required to obtain a cumulative mean bearing having the desired standard deviation may then be determined from curves such as Figs. 26-30.

If the standard deviation versus time is plotted on log-log paper, the slope of the smoothed curve is approximately minus one-half. Hence it may be inferred that the standard deviation is inversely proportional to the square-root of the time -- a fact that is stated by the central limit theorem of statistics (See Fig. 31)

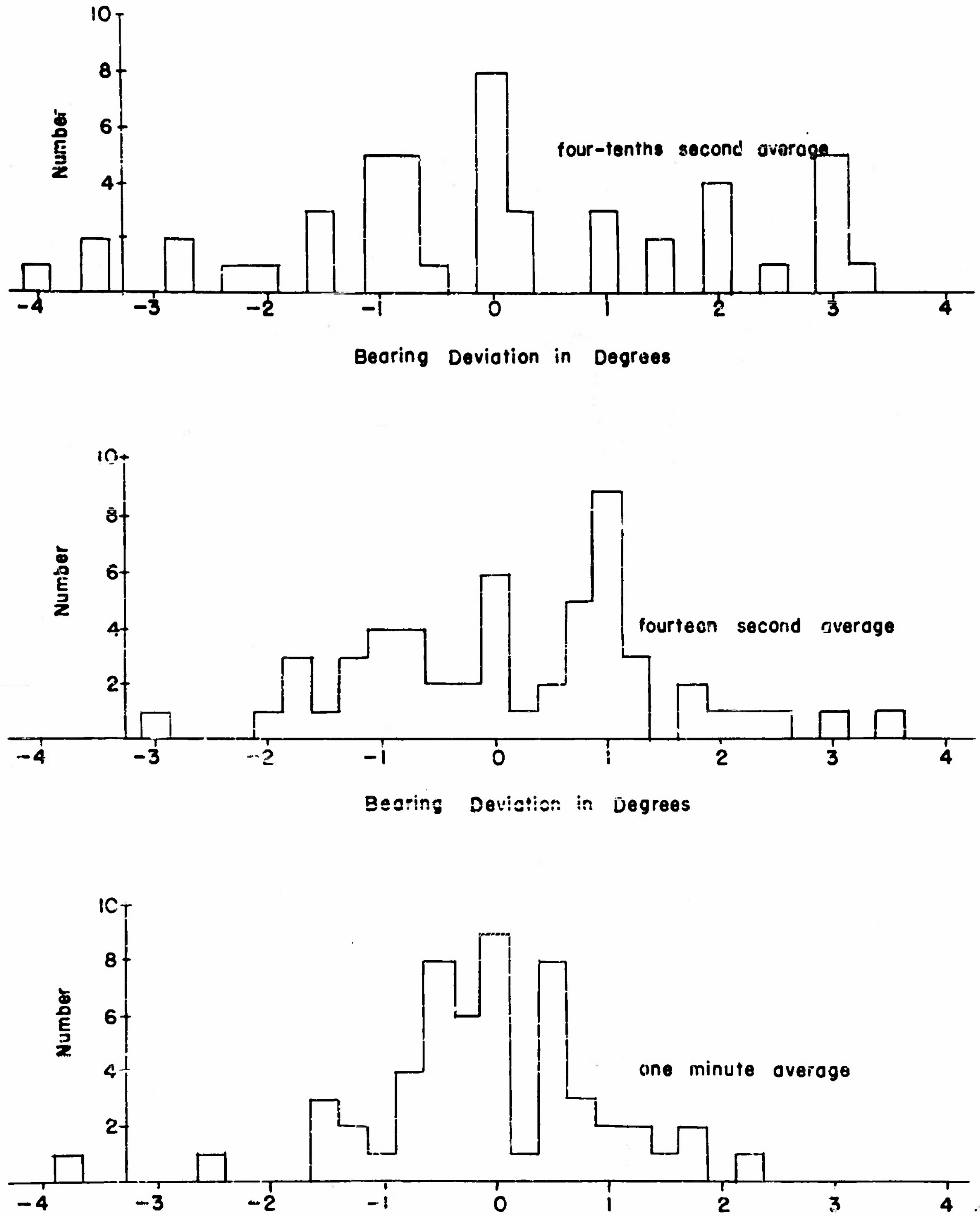

FIGURE 24 HISTOGRAMS OF CUMULATIVE MEAN BEARINGS.

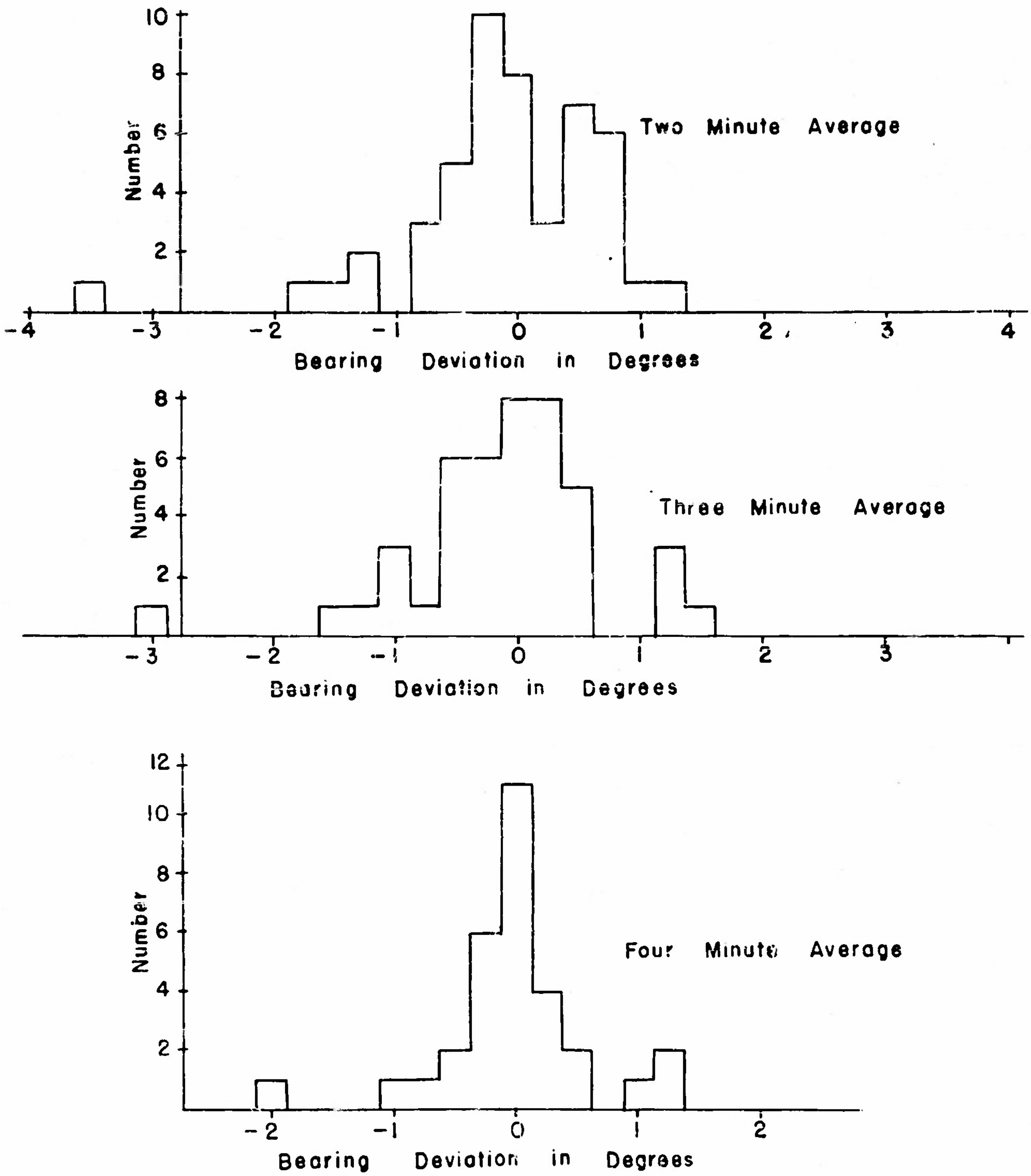

FIGURE 25 HISTOGRAMS OF CUMULATIVE MEAN BEARING FOR SELECTED AVERAGING PERIODS.

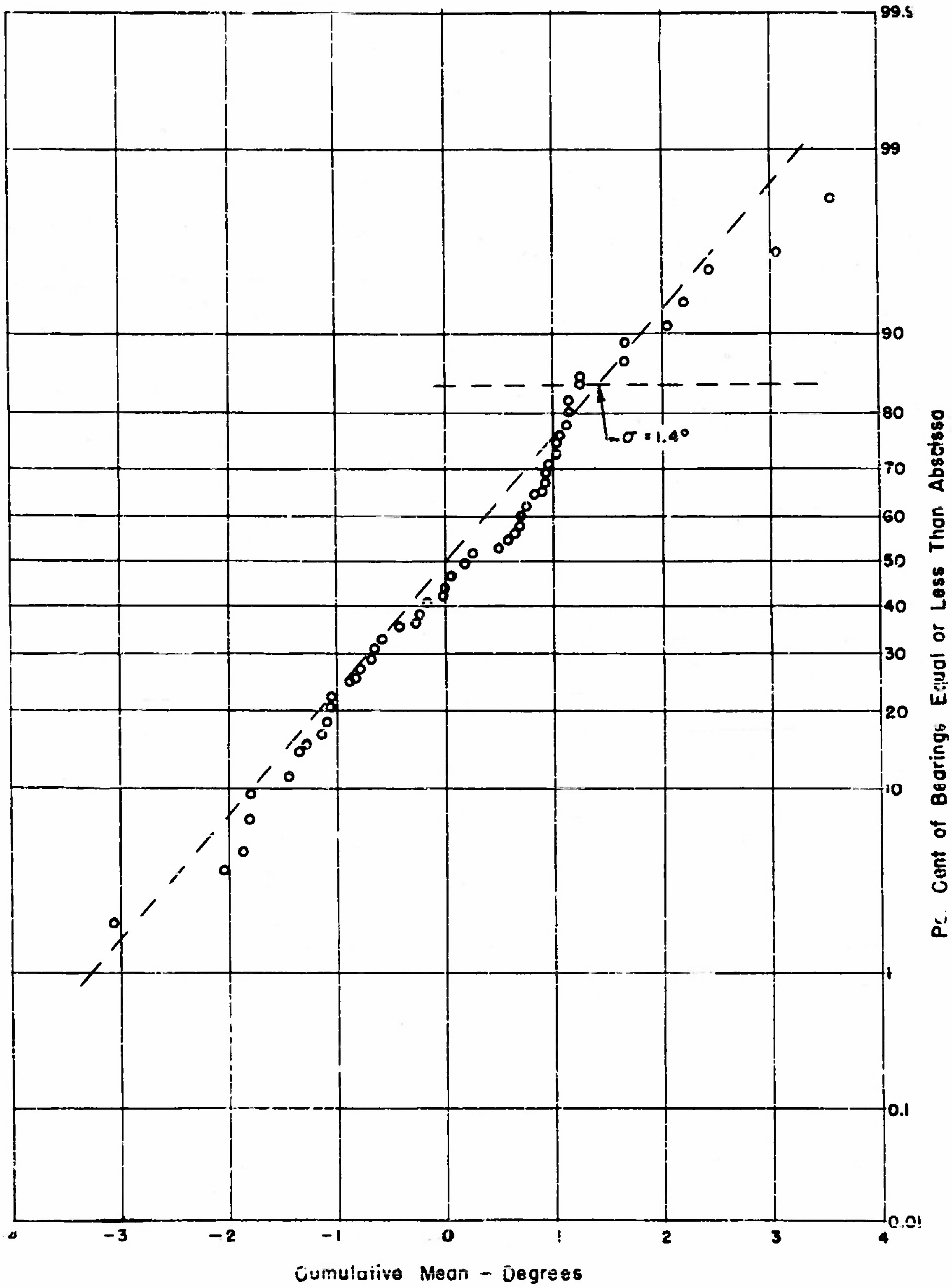

FIGURE 26 CUMULATIVE PROBABILITY DISTRIBUTION OF THE CUMULATIVE MEAN BEARING FOR A FOURTEEN SECOND SAMPLING PERIOD

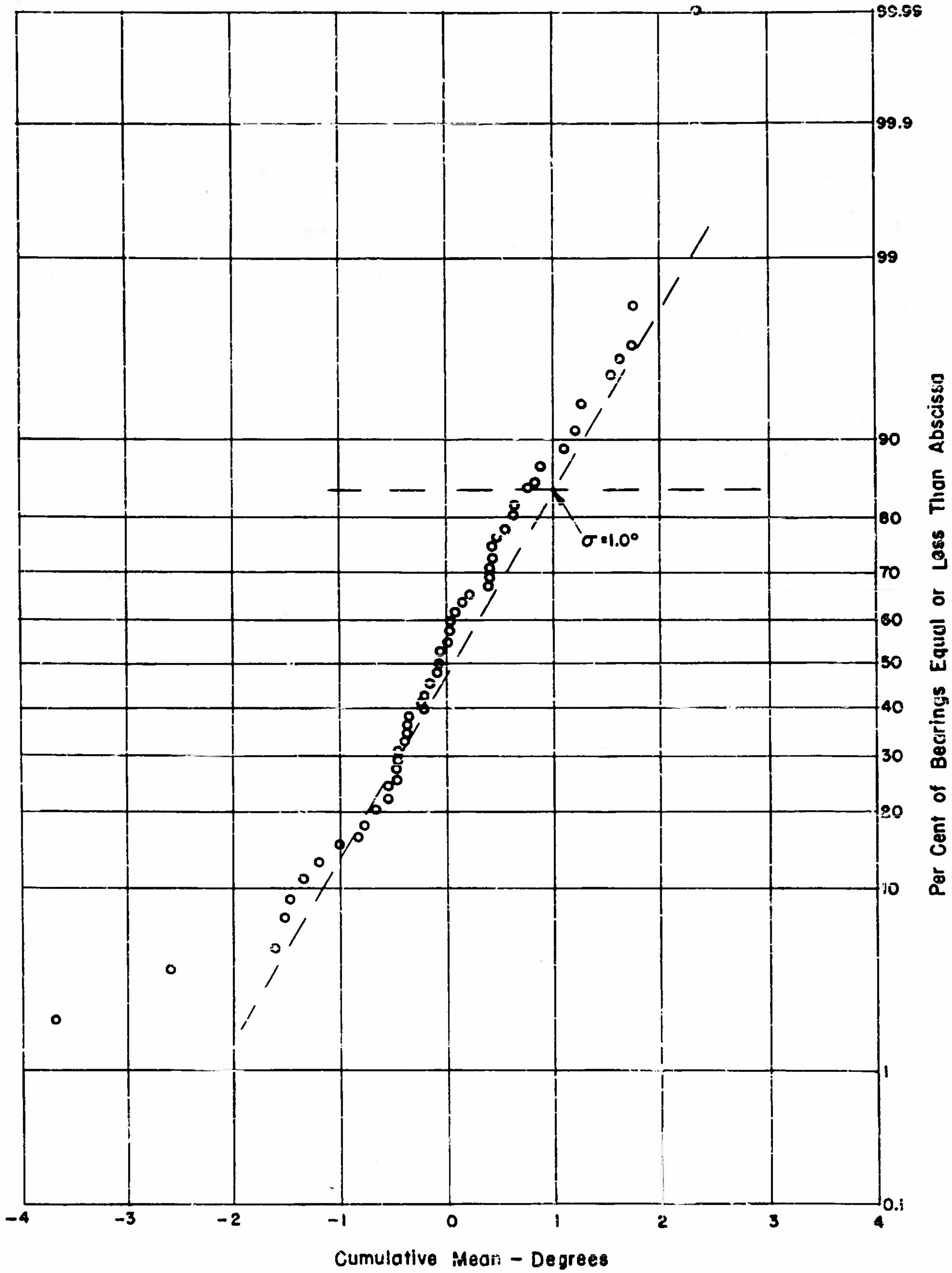

FIGURE 27 CUMULATIVE PROBABILITY DISTRIBUTION OF THE CUMULATIVE MEAN BEARING FOR A ONE MINUTE SAMPLING PERIOD

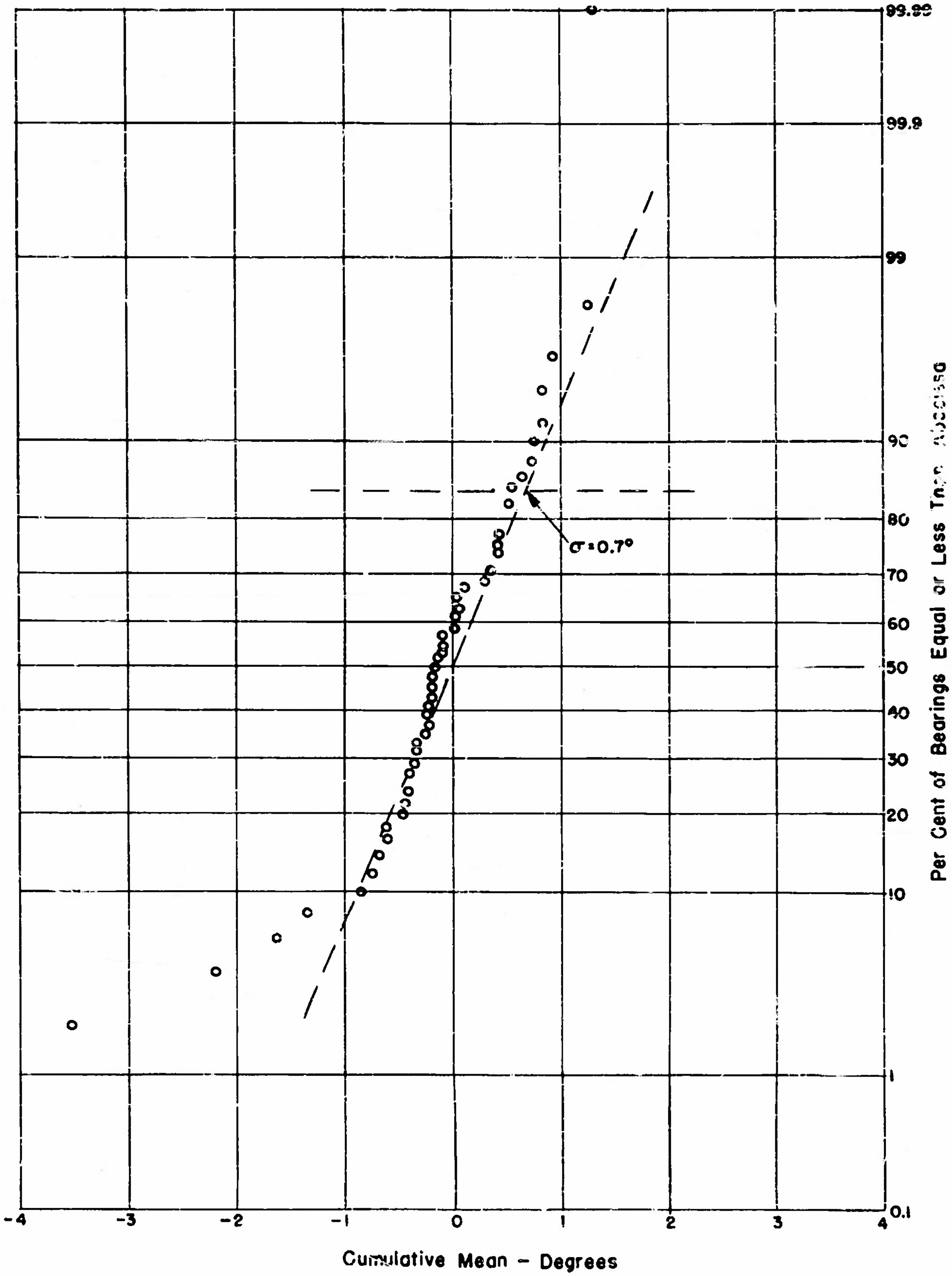

FIGURE 28 CUMULATIVE PROBABILITY DISTRIBUTION OF THE CUMULATIVE MEAN BEARING FOR A TWO MINUTE SAMPLING PERIOD

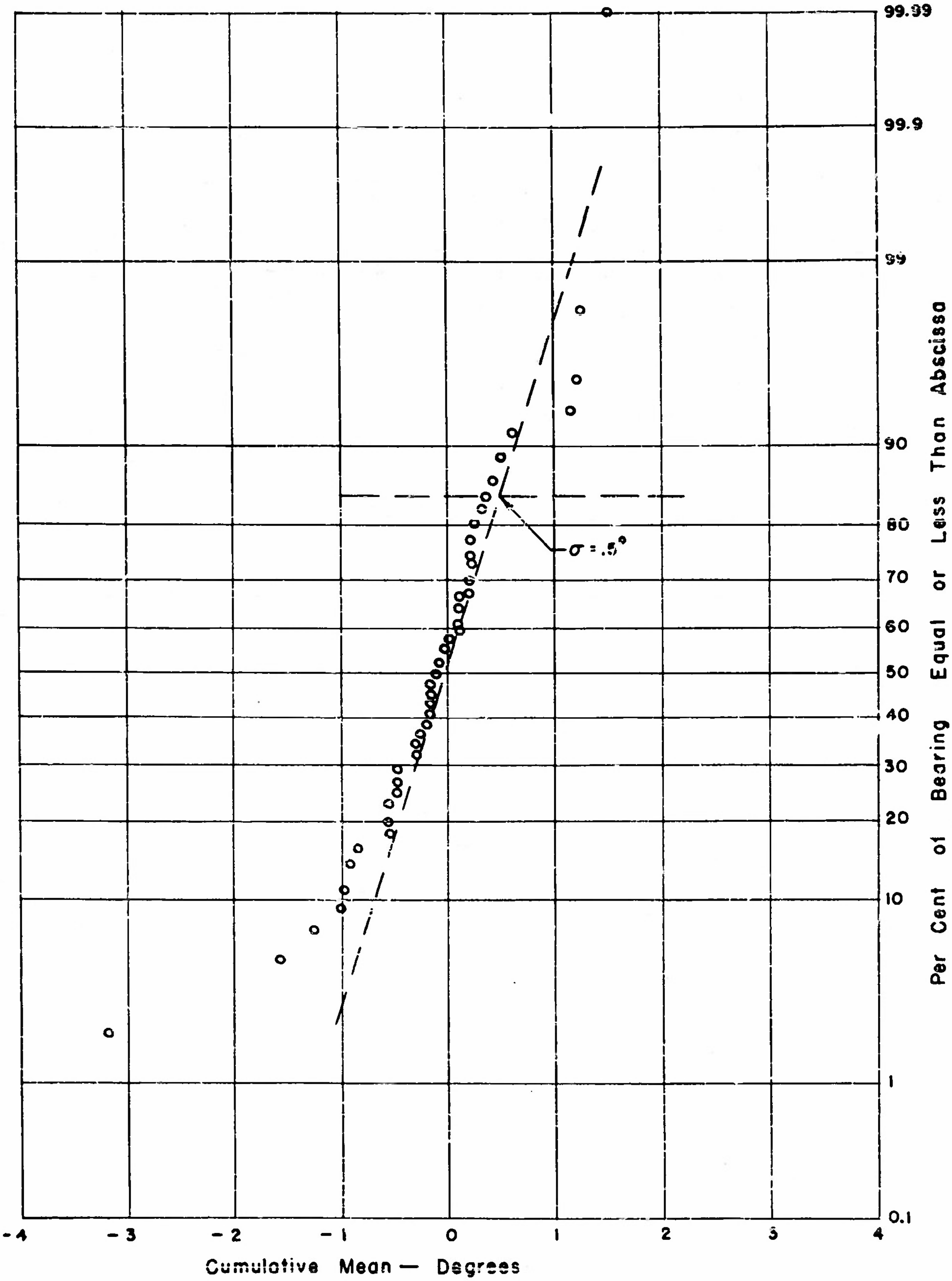

FIGURE 29 CUMULATIVE PROBABILITY DISTRIBUTION OF THE CUMULATIVE MEAN BEARING FOR A THREE MINUTE SAMPLING PERIOD

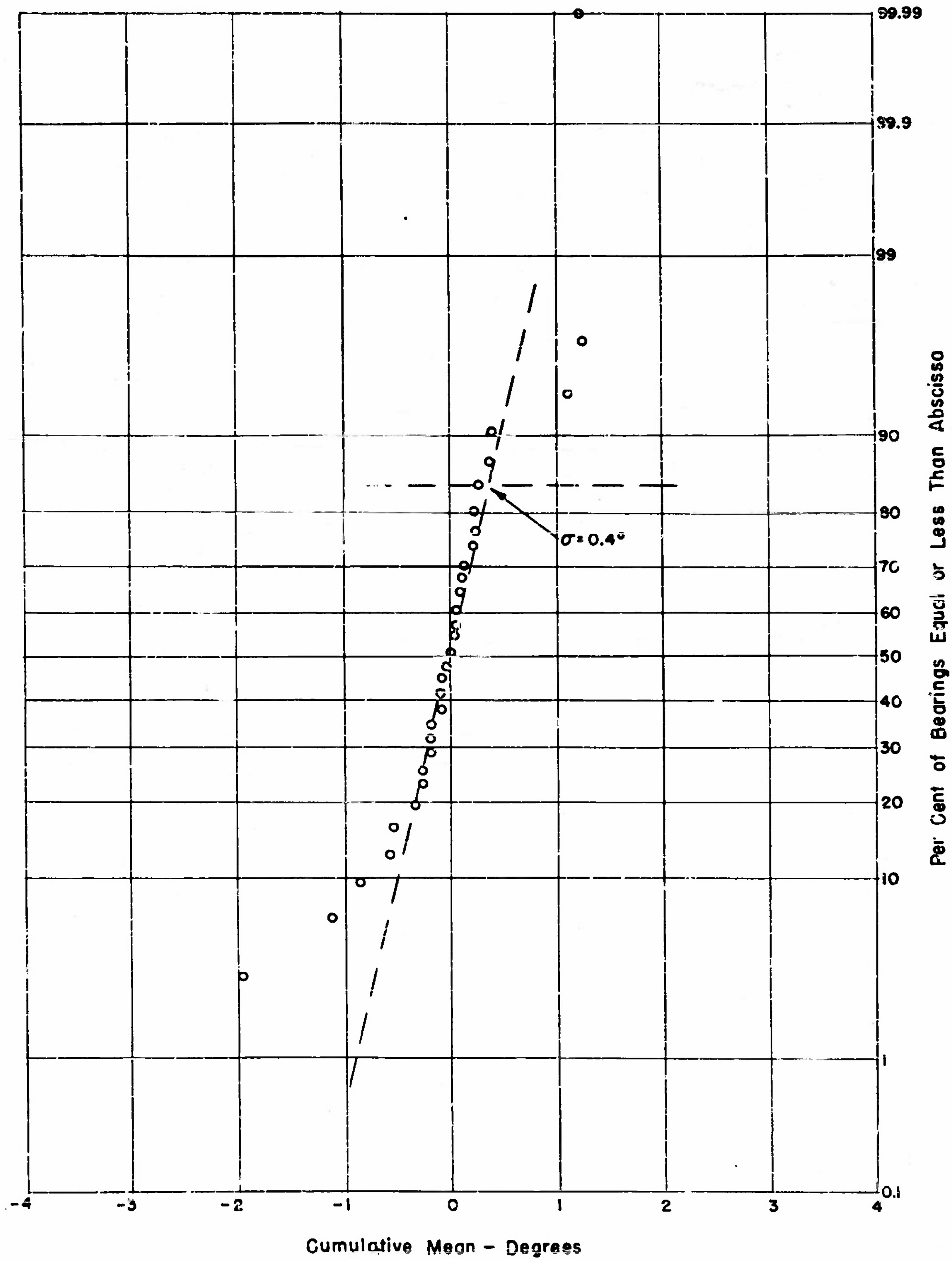

FIGURE 30 CUMULATIVE PROBABILITY DISTRIBUTION OF THE CUMULATIVE MEAN BEARING FOR A FOUR MINUTE SAMPLING PERIOD

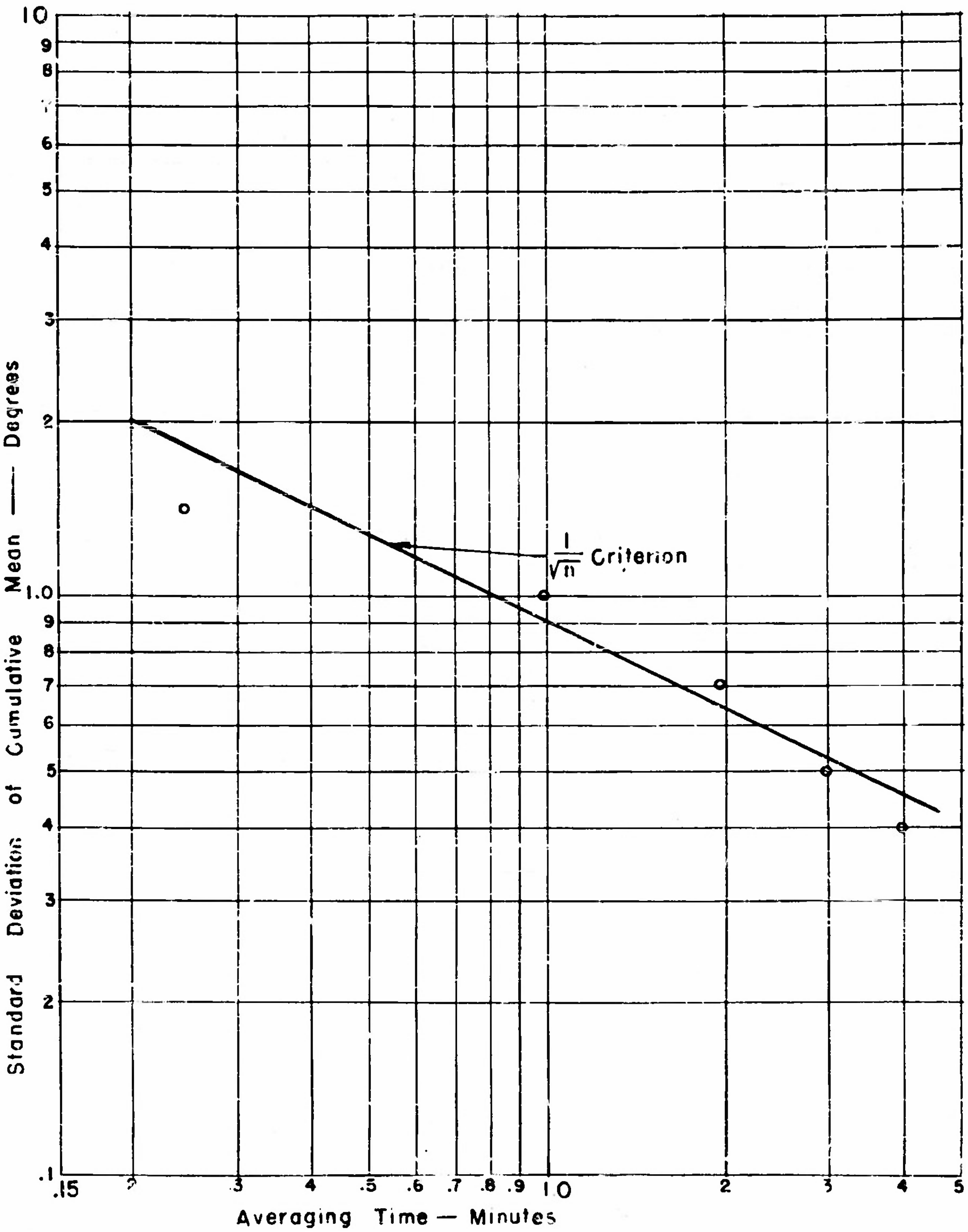

FIGURE 31 PERTINENT TO THE COMPARISON OF THE STANDARD DEVIATION OF THE CUMULATIVE MEAN VERSUS AVERAGING TIME AND A LINE EXHIBITING CENTRAL LIMIT THEOREM BEHAVIOR

7. COMPARISONS AND IMPLICATIONS

It is interesting to compare the standard deviation figures obtained in this investigation with pertinent results of other investigators.

Wilhelm Crone, a German scientist, investigated the limiting accuracy of direction of arrival measurements over a North-South path having a length of 1300 kilometers.[31] He states an accuracy figure of ±0.8° for 70% of all bearings obtained at 6 mc/s. This figure can be considered a conservative estimate of the standard deviation if normality is assumed.

W. Ross, an English scientist, investigated the limiting accuracy of direction of arrival measurements over a 920 kilometer path.[32] Whereas Crone had used a two wavelength separation between the elements of his collector, (the KOMET), Ross used a spaced-loop system having a loop spacing of 3 meters. The pertinent results of Ross compare very favorably with those of Crone. Ross concludes that both systems were measuring the deviations in the direction of arrival of radio waves. The source of these deviations is the agency that limits the short-time accuracy of any radio direction finder, whether it has a small or large aperture.

Gleason and Trexler, American scientists at the Naval Research Laboratory, made some measurements of the limiting accuracy of the direction of arrival of radio waves.[33] A standard deviation of 0.7 is quoted for the bearing error based upon all observations. The path length was 1200 nautical miles and the frequency was 6.42 mc/s. The equipment had an aperture of 1100 feet.

One implication that may be drawn from the above accuracy statements is the fact that short-time bearings of medium distance, ionosphere-reflected transmissions in the 5-10 mc/s range exhibit a standard deviation of approximately 3/4 of a degree.

The results of the present investigation indicate that a two-to-three minute cumulative average of all Adcock bearings obtained at a 25 bearing per second rate will exhibit comparable results.

31. Wilhelm Crone, *Possibilities and Limitations of Direction Finding with Sky Waves*. Paper No. 4 of Department of Scientific and Industrial Research, Special Report No. 21, entitled "Radio Direction-Finding and Navigational Aids", (London: HMSO 1951), p. 54.

32. W. Ross, *Lateral Deviation of Radio Waves Reflected at the Ionosphere*. Special Report No. 19 of the Department of Scientific and Industrial Research (London: HMSO 1949), p. 23.

33. R. F. Gleason and J. H. Trexler, *Ionospheric Limitations in the Ultimate Accuracy of Direction Finding*. NRL Memorandum Report No. 61, Naval Research Laboratory, Washington, D. C., 1952. p. 10. (CONFIDENTIAL)

The four-minute cumulative average figure of 0.4° appears to be "too good". There are at least two reasons for this. It is reasonable to expect that there will be some averaging of the long-time lateral deviation effects and this would contribute to a reduction of the standard deviation of the bearing with time. It is also logically possible that the data are not completely representative of all of the situations that may arise. (The study is continuing in order that the significance of the latter possibility may be assessed.)

Since time-averaging of the indicated bearings is so important, it is necessary to specify this along with accuracy figures of any direction finder - particularly if meaningful comparisons are to be made. From time to time, so called figures-of-merit have been proposed for radio direction finder systems. Recently the standard deviation (or its reciprocal) has come into common usage. It is suggested that a factor involving the standard deviation and the time of averaging would be a better figure-of-merit (perhaps the reciprocal of the product of the standard deviation and the averaging time).

In connection with long-time averaging of the bearings, statistical tables I and II have been prepared to show the minimum number of independent bearings required to obtain a prescribed degree of confidence in the bearing. The first table is the more pessimistic because, as is well known, the Tschebyscheff Inequality is applicable to any random variable that has finite population parameters. Either table indicates that considerably more data should be taken than is presently the case if more accurate results are desired. In the past, the emphasis has always been directed toward getting instantaneous bearings. However, the results of this investigation indicate that a small-base system can best do its job when a sufficient averaging or integrating time is permitted. Evidently we have been trying to use a "long time constant" device to do the job of a "short time constant" device.

The German Wullenweber is an example of a wide-base sector-type direction finder that was conceived and built during World War II. An experimental investigation of the system at Skisby, North Jutland, Denmark by British scientists of the ASRE yielded statistical accuracy figures that were not better than those for an Adcock group of 3 or 4 direction finders.[34] The standard deviation was found to be a function

34. A. H. Mugridge and P. G. Redgment, *The Wullenweber, The Theory, Design and Experimental Investigation of the Ex-German Wide Aperture H. F. D. F. Wullenweber at Skisby, North Jutland, Denmark*. A.S.R.E. Monograph 806, A.S.R.E. Lythe Hill House, Haslemere, Surrey, September, 1949. pp. 50-53. (RESTRICTED)

TABLE I

MINIMUM NUMBER OF SAMPLE BEARINGS REQUIRED TO OBTAIN THE STATED PROBABILITY OF A SAMPLE MEAN WITHIN ONE DEGREE OF THE TRUE MEAN FOR SEVERAL POPULATION VARIANCES - TSCHEBYSCHEFF INEQUALITY ASSUMED*+‡

Bearing Population Variance	Probability			
	0.50	0.90	0.95	0.99
$(1°)^2$	2	10	20	100
$(2°)^2$	8	40	80	400
$(3°)^2$	18	90	180	900
$(4°)^2$	32	160	320	1600
$(5°)^2$	50	250	500	2500

* Multiply tabular values by 4/9 to obtain the minimum number of bearings required if the conditions of the Camp-Meidell Inequality are known to apply.

\+ Multiply tabular values by four to find minimum number of samples required to obtain the probability of a sample mean within 1/2° of the true mean.

‡ To the nearest integer.

TABLE II

MINIMUM NUMBER OF SAMPLE BEARINGS REQUIRED TO OBTAIN THE STATED PROBABILITY OF A SAMPLE MEAN WITHIN ONE DEGREE OF THE TRUE MEAN FOR SEVERAL BEARING POPULATION VARIANCES - NORMAL LAW ASSUMED.*+

Bearing Population Variance	Probability			
	0.50	0.90	0.95	0.99
$(1°)^2$	1	3	4	7
$(2°)^2$	2	11	16	27
$(3°)^2$	4	24	35	60
$(4°)^2$	7	43	62	106
$(5°)^2$	11	68	96	166

* To nearest integer.

\+ Multiply tabular values by 4 to find minimum number of sample bearings required to obtain the probability of a sample mean with 1/2° of the true mean.

of frequency; three degrees is the approximate value based on a sequence of snap bearings obtained at 30 second intervals. The system exhibited a ten-fold increase in sensitivity over the Adcock system, was almost completely free of polarization error, and possessed very good bearing stability. The bearing stability suggests that if a small amount of time averaging had been made, a very significant reduction in the standard deviation would have been observed. Because the active elements of the array comprised a base line of approximately 70 meters, it is very likely that space averaging of the wave interference errors was not as complete as in the case of KOMET installation. However, Trexler relates a claim made ie German scientists to the effect that high frequency transmitters operating in the United States were located to within ±15 miles of their true position.[35] Trexler estimated the bearing accuracy to be within ±15 minutes.

A different viewpoint on the value of time averaging is presented by Peat in a comparison of the performance of a bearing recorder with that of a human observer.[36] The averaging time was restricted to one minute. Data were taken at ten minute intervals. Peat found that the recorder was only slightly better than a human operator. It would be of interest to know whether or not a longer averaging time, say 3 or 4 minutes, would make a marked difference in relative performance.

35. Trexler, J.H., *Circularly Disposed Antenna Arrays*, NRL Report No. R-3213, p. 19, (Washington; Naval Research Laboratory, 1949) (CONFIDENTIAL.)

36. J. D. Peat, "The Influence of the Human Element in Direction Finding". *The Marconi Review, No. 90, Vol. XI*, 1948. p. 69.

8. CONCLUSIONS

It has been shown from theoretical considerations that the effects of wave interference error in the two signal case can be reduced to zero by averaging the error function over all possible time phases. An argument was given to substantiate the same conclusion for the case of N signals. Assuming that polarization error effects are random in time, this latter error can also be reduced, in theory at least, by time averaging.

It has been demonstrated experimentally that the dispersion of the bearing error of a high quality, high frequency, twin-channel type cathode-ray direction finder may be steadily decreased by averaging for a sufficient period of time over all of the indicated bearings that are obtained at a rate that is rapid enough to preserve the instantaneous continuity of the indicated bearing. A two minute cumulative average yielded a standard deviation of approximately 0.7 of a degree, a three-minute average yielded a standard deviation of approximately 0.5 of a degree, and a four-minute average yielded a standard deviation of approximately 0.4 degree. All of these values are less than one would expect from lateral deviation effects alone. But since the lateral deviation effects discussed earlier by Ross were obtained on a north-south path, whereas these data were obtained on an east-west path which is probably less susceptible to lateral deviation effects, the results may be assumed to be approaching the ultimate limit in the determination of the direction of arrival of high frequency sky-reflected waves. Evidently the long time average not only reduced wave interference error effects but also polarization effects.

Bearing error reduction schemes that are premised upon censoring bearings according to a "strong" or "strong and steady" criterion will generally reveal an improvement in the accuracy of a bearing determination, but the improvement is not as significant as that of the cumulative bearing mean.

Because the cumulative mean bearing deviation follows the normal distribution law, Table II may be applied in predicting the number of samples and the length of time required for sampling.

APPENDIX A

A NOTE ON THE ELECTRICAL RECORDING OF ANGULAR DISPLACEMENTS

Consider the problem of recording an angular displacement that is determined by the arc tangent of a voltage ratio. Figure 32 shows the comparative plots on a linear scale of tangent ϕ and log tangent ϕ for 90 degrees of argument. If one needs to record deviation angles within ±15 degrees of a zero reference, it is evidently better to record the voltage ratio directly since there is almost a straight line relationship between deviation angle and ratio. On the other hand, if one needs to record deviation angles within ±15 degrees of a 45 degree reference it is better to record the logarithm of the voltage ratio since there is almost a straight line relationship between deviation angle and ratio. It is also significant to note that the tangent function in the case of the former and the logarithm of the tangent function in the case of the latter has a sign change at the reference value. Hence one may always infer whether or not the measured deviation is greater or less than the reference value. This makes for considerable simplicity of circuitry compared to recording deviation by the less desirable technique in each case.

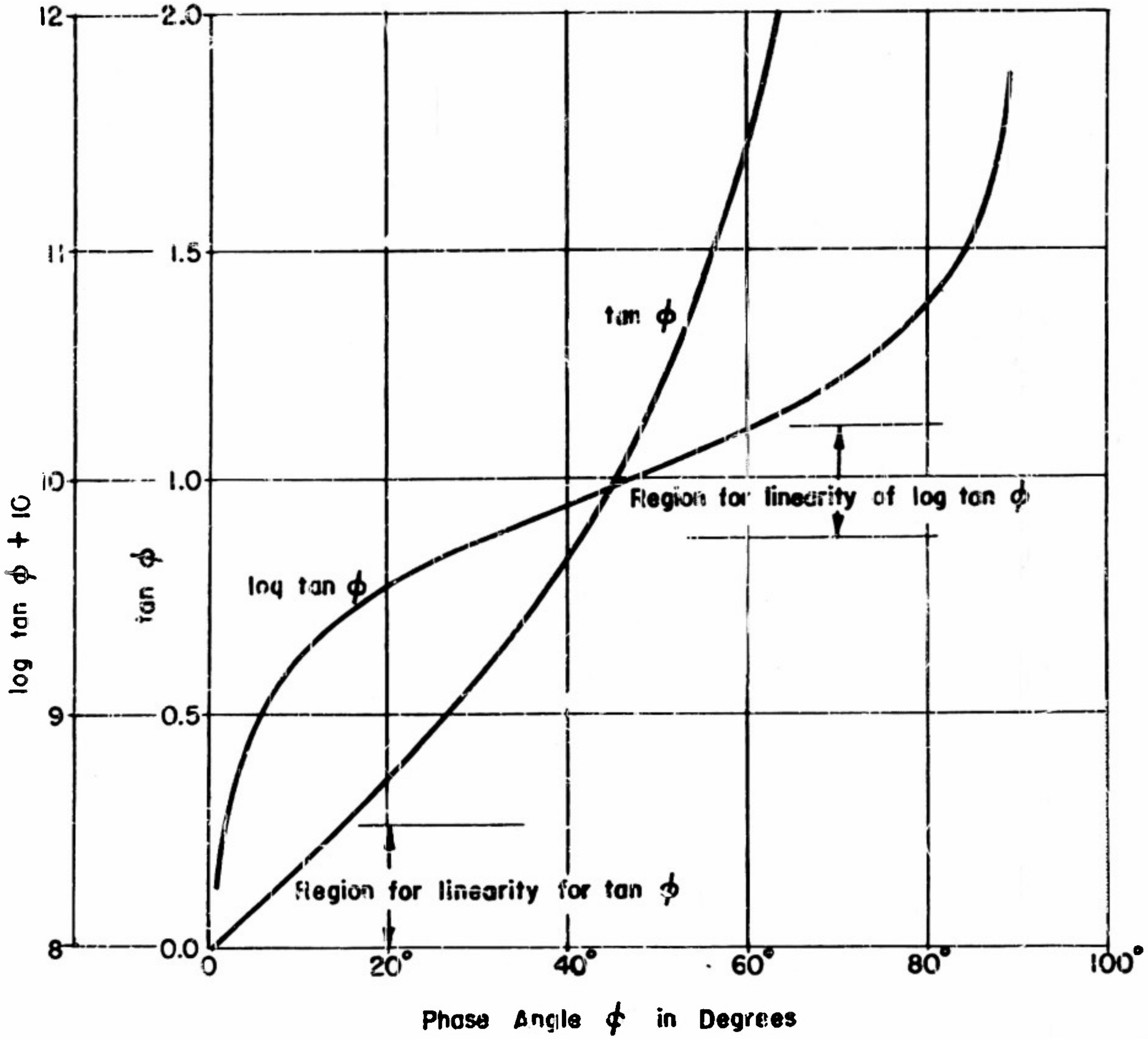

FIGURE 32 COMPARISON OF THE TANGENT AND LOGARITHM OF THE TANGENT FUNCTIONS OF ARGUMENT PHI FOR LINEARITY PURPOSES.

APPENDIX B

A NOTE ON BEARING ERROR DUE TO ELLIPSING

The bearing data that are displayed upon the indicator of a twin-channel cathode-ray direction finder may appear ideally as straight lines but more generally as ellipses. The ellipsing is due to the effects of channel misphasing, wave polarization, and wave interference. In any event the major diagonal of the ellipse is generally assumed to be the best estimate of the indicated bearing.

A compromise had to be made in the circuitry that was used for computing the bearing data that were obtained in this investigation. It was found convenient to record the inclination angle of the diagonal of the rectangle that encloses the bearing ellipse as an estimate of the indicated bearing. Because the above rectangle has its sides parallel to the X' and Y' axis of the cathode-ray tube geometry, an instrumental error will generally occur with the onset of ellipsing. Figure 33 exhibits the extremes in bearing error due to ellipsing. Figure 33a is an example of a case of maximum error, Fig. 33b is an example of a case of zero error, and between these two lie an infinity of cases.

If it were possible to orient the direction finder array or otherwise affect the bearing data so that the apparent direction of arrival of the signal as seen by the computer was in the neighborhood of 45 degrees or an odd integer multiple thereof, the ellipsing error would be minimized. A quantitative expression for the instrumental error due to ellipsing of the bearing as a function of the angle of arrival and minor-to-major axis ratio will now be determined.

The expression for an ellipse having the geometry of Fig. 33a is given by

$$b^2 X'^2 + a^2 Y'^2 = a^2 b^2, \quad \text{where } a \text{ and } b \text{ are the major and minor semi-axis, } c \text{ and } -c \text{ are the foci, and } a^2 - b^2 = c^2. \tag{1}$$

Figure 34 shows the transformation geometry for the rotation of the reference frame through an angle ϕ. The equations of transformation are

$$X' = X \cos\phi - Y \sin\phi \tag{2}$$

and

$$Y' = X \sin\phi + Y \cos\phi \tag{3}$$

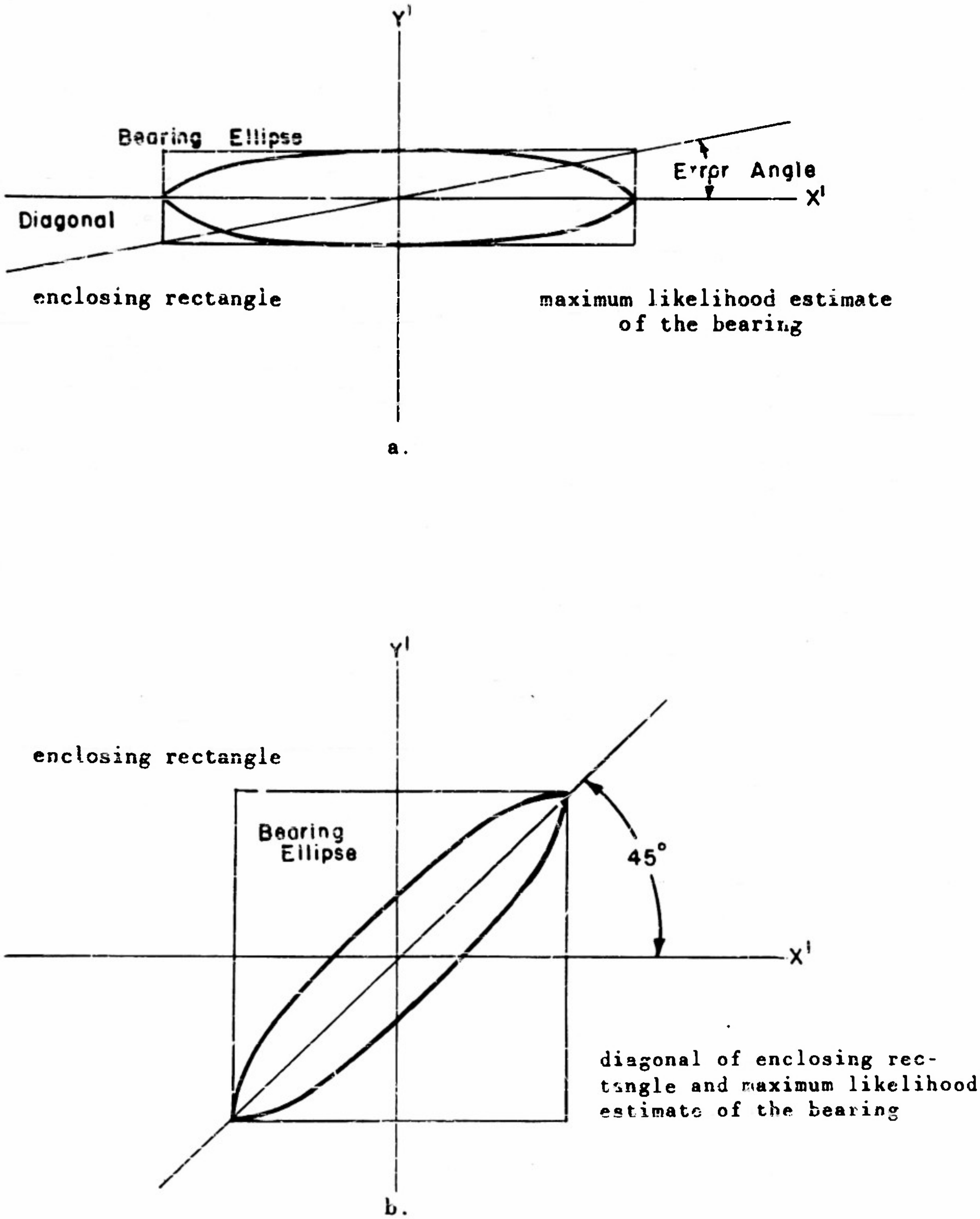

FIGURE 33 PERTINENT TO THE BEARING ERROR DUE TO ELLIPSING.

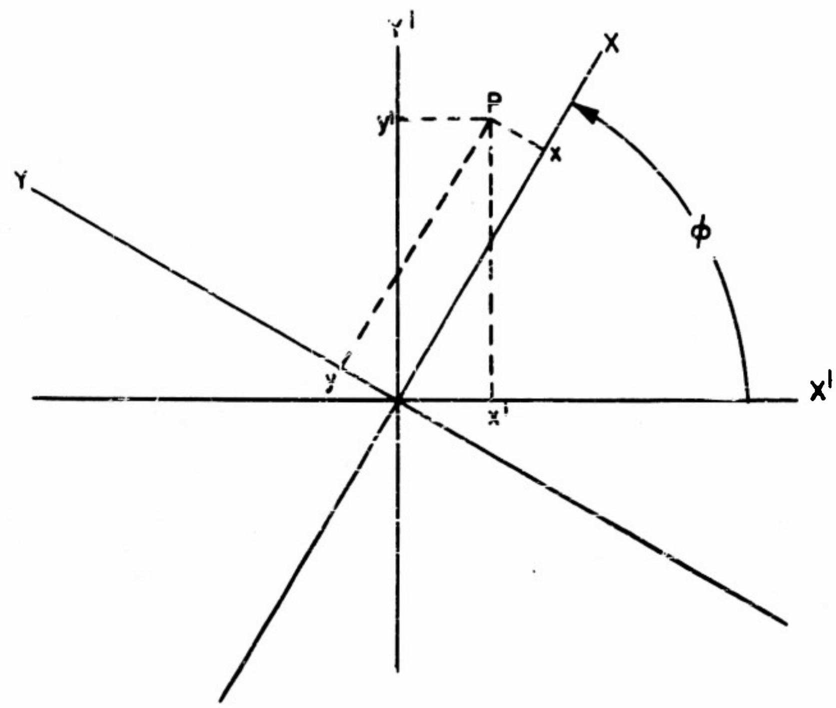

FIGURE 34 TRANSFORMATION OF COORDINATES BY ROTATION OF AXES.

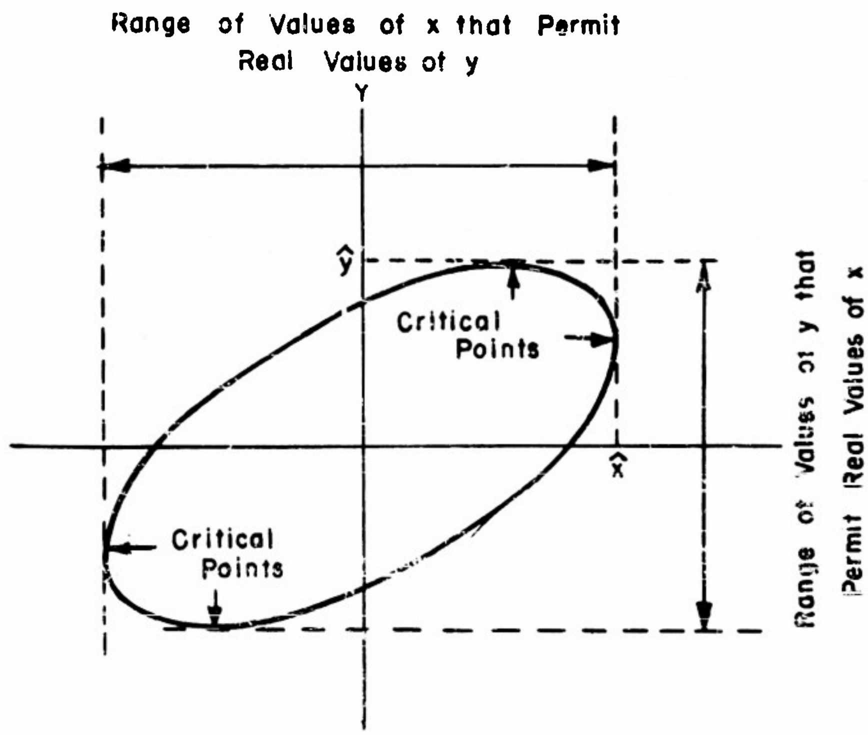

FIGURE 35 PERTINENT TO THE MEASUREMENT OF THE ANGLE OF INCLINATION OF AN ELLIPSE.

Substituting equations (2) and (3) into (1) there is obtained the transformed equation

$$X^2[a^2 \sin^2\phi + b^2 \cos^2\phi] + 2XY\,[a^2 - b^2] \sin\phi \cos\phi + Y^2\,[a^2 \cos^2\phi + b^2 \sin^2\phi] - a^2b^2 = 0 \tag{4}$$

Equation (4) can be solved explicitly for X in terms of Y and the other parameters by means of the quadratic formula. Thus

$$X = \frac{-2Y\,(a^2 - b^2) \sin\phi \cos\phi \pm \sqrt{4Y^2\,(a^2 - b^2) \sin^2\phi \cos^2\phi - 4\{Y^2(a^2 \cos^2\phi + b^2 \sin^2\phi) - a^2b^2\}\{a^2 \sin^2\phi + b^2 \cos^2\phi\}}}{2(a^2 \sin^2\phi + b^2 \cos^2\phi)} \tag{5}$$

In a similar manner

$$Y = \frac{-2X\,(a^2 - b^2) \sin\phi \cos\phi \pm \sqrt{4X^2\,(a^2 - b^2) \sin^2\phi \cos^2\phi - 4\{X^2(a^2 \cos^2\phi + b^2 \sin^2\phi) - a^2b^2\}\{a^2 \sin^2\phi + b^2 \cos^2\phi\}}}{2(a^2 \sin^2\phi + b^2 \cos^2\phi)} \tag{6}$$

In the computer circuitry, the magnitude of the maximum values of X and Y are used to determine the angle of inclination of the diagonal of the rectangle that encloses the ellipse. The immediate problem to be solved is that of finding the magnitude of the maximum values, $\hat{X}$ and $\hat{Y}$. Evidently X is real or imaginary depending upon the algebraic sign of the expression under the radical, and the value of Y that makes the expression under the radical zero is just the value of Y that determines $\hat{X}$. A similar argument applies to $\hat{Y}$. The reasons for this can be seen by examining Fig. 35. Equating the expressions under the radicals of (5) and (6) to zero and simplifying it can be shown that $\hat{X}$ is obtained when

$$Y = \pm \frac{\sqrt{a^2 \sin^2\phi + b^2 \cos^2\phi}}{\sin^2\phi + \cos^2\phi} \tag{7}$$

and $\hat{Y}$ is obtained when

$$X = \pm \frac{a^2 \cos^2\phi + b^2 \sin^2\phi}{\sin^2\phi + \cos^2\phi} \tag{8}$$

Substitution of equations (7) and (8) into equation (5) and (6), respectively, yields

$$\hat{X} = \frac{(a^2 - b^2) \sin\phi \cos\phi}{(\sin^2\phi + \cos^2\phi)\sqrt{a^2 \sin^2\phi + b^2 \cos^2\phi}} \tag{9}$$

and

$$\hat{Y} = \frac{(a^2 - b^2) \sin\phi \cos\phi}{(\sin^2\phi + \cos^2\phi)\sqrt{a^2 \cos^2\phi + b^2 \sin^2\phi}} \tag{10}$$

The tangent of the angle of inclination of the diagonal of the enclosing rectangle is given by

$$\frac{\hat{Y}}{\hat{X}} = \sqrt{\frac{a^2 \sin^2\phi + b^2 \cos^2\phi}{a^2 \cos^2\phi + b^2 \sin^2\phi}} = \sqrt{\frac{\tan^2\phi + (\frac{b}{a})^2}{1 + (\frac{b}{a})^2 \tan^2\phi}} \tag{11}$$

The expression for the ellipsing error may now be written as

$$\varepsilon = \arctan \sqrt{\frac{\tan^2\phi + (\frac{b}{a})^2}{1 + (\frac{b}{a})^2 \tan^2\phi}} - \phi \tag{12}$$

It is evident that when ϕ = 45 or odd integer multiples thereof, the error is zero, and when ϕ = 0 or even multiples of 45 degrees the error angle is arc tangent $(\frac{b}{a})$ or arc tangent $(\frac{a}{b})$ depending upon the integer.

The family of curves shown in Fig. 36 has been calculated to show instrumental error due to ellipsing as a function of the angle of arri-

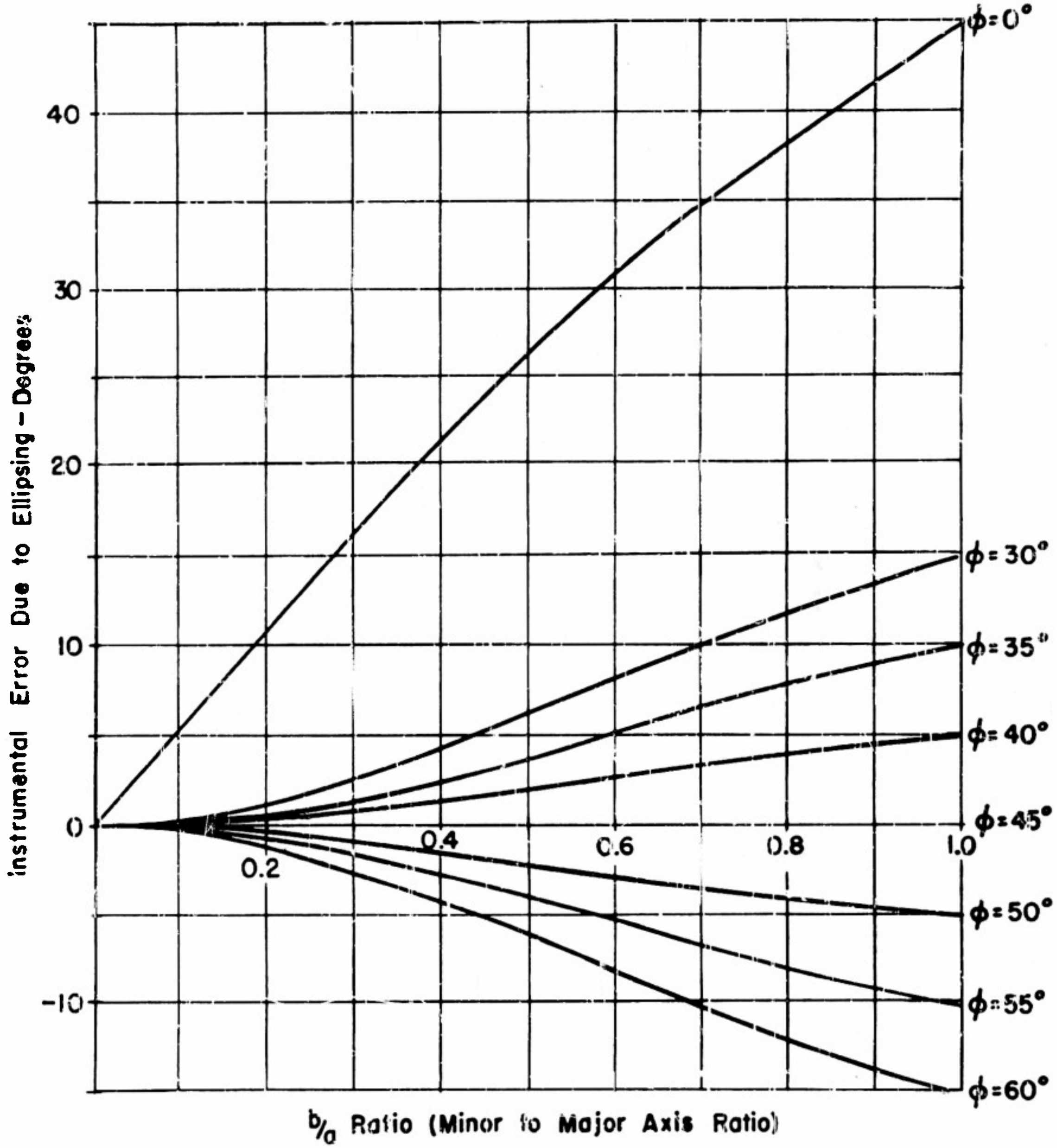

FIGURE 36 ELLIPSING ERROR IN BEARING COMPUTER AS A FUNCTION OF b/a RATIO AND THE MEAN AZIMUTH

val ϕ, and the minor-to-major axis ratio. It is seen that even for small values of ellipsing corresponding to minor-to-major axis ratio of say 0.1, the error may be greater than 5 degrees if the signal is arriving from 0 degrees or even integer multiples of 45 degrees. However, if the signal is arriving from odd integer multiples of 45 degrees there is no error, and the error does not increase appreciably for moderate amounts of ellipsing provided that the angle of arrival is within ±15 degrees of the optimum angle of arrival.

An important observation to be drawn from the above discussion is that in radio direction finding techniques that utilize the ratio of the magnitude of two voltages that are derived from orthogonally oriented differential antennas to determine the angle of arrival, it is imperative that the collector be oriented so that the mean direction of arrival of the signal is from some odd integer multiple of 45 degrees with respect to the plane of either differential antenna.

APPENDIX C

A NOTE ON THE CORRECTION FOR RESIDUAL GAIN UNBALANCES IN THE COMPUTER INPUT CIRCUITS

The transformation required to minimize the instrumental error of the computer due to ellipsing of the bearing can be rewritten as

$$|E'_H| = |E_H| \operatorname{Cos}\phi - |E_v| \operatorname{Sin}\phi$$

$$|E'_v| = |E_H| \operatorname{Sin}\phi + |E_v| \operatorname{Cos}\phi$$

where $E_H = K \operatorname{Sin} \theta$, $E_v = K \operatorname{Cos} \theta$, K is a constant for the radio direction finder, and θ is the angle of arrival measured ccw from the 90 degree azimuth reference. Evidently the ratio $|E'_v|/|E'_H| = \operatorname{Cot}(\theta - \phi)$ and by properly selecting ϕ, $\theta - \phi = (2n - 1)\pi/4$. A phase shifter comprised of two properly interconnected goniometers can provide the appropriate value of ϕ for any given θ.

In any practical equipment it is not possible to maintain identical gains in two channels for any extended period, hence in general

$$E_H = K(1 - \Delta)\operatorname{Sin}\phi$$

$$E_v = K(1 + \Delta)\operatorname{Cos}\phi$$

where Δ is one-half the incremental unbalance in gain. The ratio of $|E'_v|/|E'_H|$ $\operatorname{Cot}(\theta - \phi)$, and it becomes necessary to investigate the possibility of correcting for gain unbalance by setting in an additional bearing shift of degrees. The ratio then becomes

$$\frac{|E'_v|}{|E'_H|} \doteq \frac{\operatorname{Cos}(\theta - \phi + \delta) + \Delta \operatorname{Cos}(\theta + \phi - \delta)}{\operatorname{Sin}(\theta - \phi + \delta) + \Delta \operatorname{Sin}(\theta + \phi - \delta)}$$

Assuming that δ is small enough and that the product of δ and Δ is an infinitesimal of higher order, the ratio simplifies to

$$\frac{|E'_v|}{|E'_H|} \doteq \frac{\operatorname{Cos}(\theta - \phi) - \delta \operatorname{Sin}(\theta - \phi) + \Delta \operatorname{Cos}(\theta + \phi)}{\operatorname{Sin}(\theta - \phi) + \delta \operatorname{Cos}(\theta - \phi) - \Delta \operatorname{Sin}(\theta + \phi)}$$

Now imposing the requirement that $(\theta - \phi)$ be an odd integer multiple of 45 degrees and that at balance the ratio be unity there is obtained for the case $(\theta - \phi) = 45$ degrees

$$\frac{|E'_V|}{|E'_H|} \doteq \frac{\frac{1}{\sqrt{2}} - \delta/\sqrt{2} + \Delta \operatorname{Cos}(\theta - \phi)}{\frac{1}{\sqrt{2}} + \delta/\sqrt{2} - \Delta \operatorname{Sin}(\theta + \phi)} = 1$$

from which

$$\delta \doteq \frac{\Delta}{(\sqrt{2})} [\operatorname{Cos}(\theta + \phi) + \operatorname{Sin}(\theta + \phi)]$$

or

$$\delta \doteq \Delta \operatorname{Sin}(\theta + \phi + \frac{\pi}{4})$$

Thus it has been shown that subject to certain assumptions it is possible to set in a δ that will correct for an incremental gain unbalance of 2Δ in the amplifiers of the computer input circuits. The practical procedure for accomplishing this is to adjust the gains of the two channels as closely as is possible in order to make Δ small, and then use the bearing shifter to bring the system into the balanced condition of zero output when the source of signal is a target transmitter placed at reference zero bearing.

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